MURAKKAB ARGUMENTLI TRIGONOMETRIK TENGLAMALARNI YECHISH.

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Annotation: There are multiple standard and non-standard methods for solving trigonometric equations, and these methods can vary significantly. Solving trigonometric equations with complex arguments often presents a greater challenge for students. Even when they manage to solve the equation, they may struggle to identify the key elements of the solution. Addressing these challenges by incorporating the study of solving such equations and inequalities into both mathematics circles and faculty training is crucial.

Keywords: trigonometric equations, complex arguments, lowering the degree

1-misol. Ushbu

$$\sin(\sin(\cos x - \sin x)) = 0$$

tenglamani yeching.[1]

Yechish: $\sin(\cos x - \sin x) = \pi n, n \in Z$. $-1 \le \sin t \le 1$ bo‘lgani uchun $-1 \le \pi n \le 1$ bo‘lishi kerak. Bundan $n = 0$. Endi $\sin(\cos x - \sin x) = 0$ tenglamani yechamiz. Bu $\cos x - \sin x = \pi k, k \in Z$ tenglamaga teng kuchli.

$\sin(x - \frac{\pi}{4}) = \frac{\sqrt{2}\pi k}{2}$. k ning qiymati $-1 \le \frac{\pi k}{\sqrt{2}} \le 1$ tengsizlikni qanoatlantirishi kerak, bundan $k = 0$. Demak, $\sin(x - \frac{\pi}{4}) = 0$ tenglamani yechamiz. $x = \frac{\pi}{4} + \pi m, m \in Z$

Javob: $\frac{\pi}{4} + \pi m, m \in Z$.

2-misol. Ushbu

$$\cos(\pi\sqrt{1-\sin x})=1$$

tenglamani yeching. [1]

Yechish: $\sqrt{1-\sin x}=2k, k\in N_0$ bo‘lishi kerak.U holda $sinx=1-4k^2$, $-1\le \sin x\le 1$ bo‘lgani uchun $-1\le 1-4k^2\le 1$, $-2\le -4k^2\le 0$; $0\le k^2\le \frac{1}{2}$. Demak, $k=0$ va $sinx=1$.

Javob: $\frac{\pi}{2}+\pi n, n\in Z$.

3-misol. Ushbu

$$\sin^2(1-\cos x)=\cos^2(1+\cos x)$$

tenglamani yeching. [2]

Yechish. Darajani pasaytirish formulasini qo‘llab

$$cos(2+2\cos x)+\cos(2-2\cos x)=0$$

tenglamani hosil qilamiz. Kosinuslar yig’indisini ko‘paytmaga o‘tkazib $\cos(2\cos x)=0$; $cosx=\frac{\pi}{4}+\frac{\pi n}{2}, n\in Z$ ni hosil qilamiz. n ning qabul qilishi mumkin bo‘lgan qiymatlarini topish uchun $-1\le\frac{\pi}{4}+\frac{\pi n}{2}\le 1$ tengsizlikni yechamiz. Bundan $n=0$ yoki $n=-1$. Demak, $|\cos x|=\frac{\pi}{4}$

Javob: $\pm\arccos\frac{\pi}{4}+\pi k, k\in Z$.

4-misol. Ushbu

$$\cos x + \cos\sqrt{x} = 2$$

tenglamani yeching. [1]

Yechish: $|cos x| \le 1$ va $|cos\sqrt{x}| \le 1$ bo‘lgani uchun berilgan tenglama quyidagi sistemaga teng kuchli

$$\begin{cases} cos x = 1 \\ \cos\sqrt{x} = 1 \end{cases}, \begin{cases} x = 2\pi k, k \in Z \\ x = 4\pi^2 m^2, m \in Z \end{cases}$$

Bundan $2\pi k = 4\pi^2 m^2; k = 2\pi m^2$, k va m lar butun son bo‘lganligi uchun oxirgi tenglik $k = m = 0$ da o‘rinli.

Javob:0

5-misol. Ushbu

$$\cos\sqrt{x} = \cos\sqrt{x+1}$$

tenglamani yeching. [2]

Yechish: Berilgan tenglama quyidagi tenglamaga teng kuchli.

$$-2sin\frac{\sqrt{x}-\sqrt{x+1}}{2}sin\frac{\sqrt{x}+\sqrt{x+1}}{2} = 0$$

Bundan $sin\frac{\sqrt{x+1}-\sqrt{x}}{2} = 0$; $sin\frac{\sqrt{x}+\sqrt{x+1}}{2} = 0$. x ni topish uchun quyidagi 2 ta tenglamani yechamiz. $\sqrt{x+1}-\sqrt{x} = 2\pi k, k \in N$; $\sqrt{x}+\sqrt{x+1} = 2\pi n, \mathrm{n} \in N$;

$x > 0$ da $\sqrt{x+1}-\sqrt{x} > 0$ bo‘lganligi uchun $k \in N$

$x > 0$ da $\sqrt{x}+\sqrt{x+1} > 0$ bo‘lganligi uchun $k \in N$.

Birinchi tenglamani yechamiz. $x+1 = x + 4\pi k\sqrt{x} + 4\pi^2 k^2;\ 4\pi k\sqrt{x} = 1 - 4\pi^2 k^2$

Bu tenglama $k \in N$ da yechimga ega emas.Ikkinchi tenglamani yechamiz. $\sqrt{x} = 2\pi n - \sqrt{x+1}$. Bu tenglama quyidagi sistemaga teng kuchli.

$$\begin{cases} x = 4\pi^2 n^2 - 4\pi n\sqrt{x+1} + x + 1; \\ 2\pi n \geq \sqrt{x+1} \end{cases}$$

Sistemaning birinchi tenglamasidan $\sqrt{x+1} = \dfrac{4\pi^2 n^2 + 1}{4\pi n}$ $\sqrt{x+1} < 2\pi n, (n \in N)$ ekanligini ko‘rsatamiz.Haqiqatan ham,

$$\frac{4\pi^2 n^2 + 1}{4\pi n} < 2\pi n;\ \frac{-4\pi^2 n^2 + 1}{4\pi n} < 0$$

Oxirgi tengsizlik $n \in N$ da o‘rinli. Demak, $x = (\dfrac{4\pi^2 n^2 + 1}{4\pi n})^2 - 1 = (\dfrac{4\pi^2 n^2 - 1}{4\pi n})^2, n \in N$

Javob: , $(\dfrac{4\pi^2 n^2 - 1}{4\pi n})^2, n \in N$

Foydalanilgan adabiyotlar.

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