$\Phi_{[0,l]}$ TO'PLAM VA UNING ASOSIY XOSSALARI.

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ANNOTATSIYA

Ushbu maqolada $\Phi_{[0,l]}$ to'plam va uning asosiy xossalari ko'rib chiqilgan.

ANNOTATION

This article examines the set $\Phi_{[0,l]}$ and its underlying properties.

Аннотация

В этой статье рассматривается множество $\Phi_{[0,l]}$ и его основные свойства.

$\Phi_{[0,l]}$ to'plam va uning asosiy xossalari.

1-ta'rif. Agar $(0,l_0]$ da aniqlangan $\varphi(\delta)$ funksiya quyidagi

a). $\lim\limits_{\delta\to 0}\varphi(\delta)=0$;

b). $\varphi(\delta)$ deyarli o'suvchi;

d). $\sup\limits_{\delta>0}\frac{1}{\varphi(\delta)}\int_0^{\delta}\frac{\varphi(t)}{t}dt=A_0<\infty$;

e). $\sup\limits_{\delta\to 0}\int_0^{l_0}\frac{\varphi(t)}{t^2}dt=B_0<\infty$;

shartlarni qanoatlantirsa, u holda $\varphi(\delta)\in\Phi_{[0,l]}$ deb ataymiz.

$\varphi(\delta)\in\Phi_{[0,l_0]}$ to'plam quyidagi xossalarga ega:

1). Agar $\varphi(\delta)\in\Phi_{[0,l_0]}$ bo'lsa, u holda $\frac{\varphi(t)}{t}$ deyarli o'suvchi;

2). Agar $\varphi(\delta)\in\Phi_{[0,l_0]}$ bo'lsa, u holda $\exists\alpha>0,\beta<1$ sonlar topilib, $\frac{\varphi(t)}{t^{\alpha}}$ deyarli o'suvchi, $\frac{\varphi(t)}{t^{\beta}}$ deyarli kamayuvchi bo'ladi;

3). Agar $\varphi(\delta) \in \Phi_{[0,l_0]}$ bo'lsa, u holda $\frac{t}{\varphi(t)} \in \Phi_{[0,l_0]}$ bo'ladi;

4). a) va b) , d) va 2) shartlarning bajarilishi ushbu

f) $\exists C > 0,\ 1 < \lim\limits_{\delta\to 0} \frac{\varphi(C\delta)}{\varphi(\delta)} \leq \lim\limits_{\delta\to 0} \frac{\varphi(C\delta)}{\varphi(\delta)} < C$

Shartlarning bajarilishiga ekvivalent bo'ladi.

Quyidagi funksiyalar $\Phi_{[0,l_0]}$ to'plamga qarashli bo'ladi:

1). $\varphi(t) = t^{\alpha}\ 0 < t < 1;$

2). $\varphi(t) = t^{\alpha}|\ln t|^{p},\ \varphi(0) = o$, bunda $0 < \alpha < 1, p > 0;$

3). $\varphi(t) = \sum_{n=1}^{\infty} K_n t^{\delta_n}$, bunda $K_n > 0, 0 < \delta_n < 1$

$$\lim_{n\to\infty} \delta_n = \delta > 0, \quad \delta_n > \delta\ \ (n = \overline{1,\infty}),\ \sum_{n=1}^{\infty} K_n < \infty;$$

4). $\varphi(t) = \begin{cases} t^{\alpha}, & \text{agar } o \leq t \leq \frac{l_0}{2}, \\ t^{\alpha} + 1, & \text{agar } \frac{l_0}{2} \leq t \leq l_0 \text{ bo'lsa.} \end{cases}$

Takidlaymizki, 4) misoldan ko'rinadiki $\Phi_{[0,l_0]}$ to'plamga qarashli bo'ladigan funksiyalar uzulishga ega, xatto monoton bo'lmasligi ham mumkin ekan.

1-teorema. Agar $\varphi(\delta) \in \Phi_{[0,l_0]}$ bo'lsa, u holda :

1. $\varphi_1(\delta) = \int_0^{\delta} \frac{\varphi(t)}{t} dt,\quad \varphi(\delta)$ ga ekvivalent bo'ladi.

2. $c = \int_0^{\delta} \frac{\varphi_1(t)}{t} dt + \int_0^{l_0} \frac{\varphi_1(t)}{t^2} dt,\quad \varphi_1(\delta)$ ga ekvivalent bo'ladi va u modul uzluksiz bo'ladi.

3. $\varphi_3(\delta) = \int_0^{\delta} \frac{\varphi_2(t)}{t} dt$, $\varphi_2(\delta)$ ga ekvivalent, modul uzluksiz bo'ladi hamda uning hosilasi $\varphi_3{}'(\delta)$, $\frac{\varphi_3(\delta)}{\delta}$ ga ekvivalent bo'ladi.

Isbot. $\varphi(\delta) \in \Phi_{[0,l_0]}$ bo'lsin. U holda b) shartga asosan $\varphi_1(\delta) \leq A_{\varphi}\varphi(\delta)$. Ikkinchi tomondan $\frac{\varphi(\delta)}{\delta}$- deyarli kamayuvchi bo'lgani uchun $\exists C_{\varphi} > 0$ son topiladiki $\forall \delta_1 < \delta_2$ lar uchun

$$\frac{\varphi(\delta_1)}{\delta_1} \geq C_{\varphi} \frac{\varphi(\delta_2)}{\delta_2}$$

tengsizlik o'rinli bo'ladi.

Demak,

$$\varphi_1(\delta) = \int_0^\delta \frac{\varphi(t)}{t} dt \geq C_\varphi \varphi(\delta)$$

bo'ladi. Shunday qilib

$$C_\varphi \varphi(\delta) \leq \varphi_1(\delta) \leq A_\varphi \varphi(\delta),$$

ya'ni $\varphi_1(\delta) \sim \varphi(\delta)$, $\varphi_1(\delta)) \in \Phi_{[0,l_0]}$ bo'ladi.

Endi ikkinchi tasdiqni isbot qilamiz. Xuddi yuqoridagi singari

$$\int_0^\delta \frac{\varphi_1(t)}{t} dt \geq C_{\varphi_1} \varphi_1(\delta)$$

ga ega bo'lamiz. $\varphi_1(\delta) -$ monoton o'suvchi bo'lganligi uchun $t < \xi < l_0$ lar uchun $\varphi_1(\xi) > \varphi_1(t)$

Shunday qilib

$$\delta \int_0^\delta \frac{\varphi_1(\xi)}{\xi^2} d\xi \geq \varphi_1(\delta) \frac{l_0 - \delta}{l_0} .$$

Bundan,

$$\varphi_2(\delta) \geq C_{\varphi_1} \varphi_1(\delta) .$$

Ikkinchi tomondan $\varphi_1(\delta)$ uchun $d)$ va $e)$ shartlarga ko'ra

$$\varphi_2(\delta) \leq A_{\varphi_1(\delta)} + B_{\varphi_1} \varphi_1(\delta)$$

ni hosil qilamiz. Yuqoridagi tengsizliklardan

$$\varphi_2(\delta) \sim \varphi_1(\delta)$$

ekanligi kelib chiqadi.

$$\left(\frac{\varphi_2(\delta)}{\delta}\right)' = -\frac{1}{\delta^2} \int_0^\delta \frac{\varphi_1(t)}{t} dt < 0, (0, l_0]$$

ekanligini e'tiborga olsak, u holda $\frac{\varphi_2(\delta)}{\delta}$ ning o'suvchi ekanligiga ishonch hosil qilamiz. Demak $\varphi_2(\delta)$- modul uzluksiz bo'lar ekan.

$\varphi_3'(\delta) = \frac{\varphi_2(\delta)}{\delta}$ va $\varphi_2(\delta) \sim \varphi_1(\delta)$ bo'lgani uchun bundan $\varphi_3(\delta) \sim \frac{\varphi_2(\delta)}{\delta}$ ekanligi kelib chiqadi. Bu esa, teoremaning o'liq isbot bo'lganligini anglatadi.

1-teoremaga asosan $\Phi_{[0,l_0]}$ to'plamning ta'rifini quyidagi ko'rinishda ham berish mumkin:

2-ta'rif. Agar $(0,l_0]$ da aniqlangan $\varphi(\delta)$ funksiya quyidagi

a). $\omega(\delta) -$ modul uzluksiz;

b). $\delta \int_{\delta}^{l_0} \frac{\omega(t)}{t(t+\delta)} dt \leq D\omega(\delta)$

d). $\omega'(\delta) \sim \frac{\varphi(\delta)}{\delta}$ shartlarni qanoatlantirsa, u holda $\varphi(\delta) \in \Phi_{[0,l_0]}$ deb ataymiz.

1-ta'rifdagi d) va e) shartlar 2-ta'rifdagi bitta

$$\delta \int_{\delta}^{l_0} \frac{\omega(t)}{t(t+\delta)} dt \leq D\omega(\delta)$$

shartga almashtiriladi, chunki

$$\int_0^{\delta} \frac{\omega(t)}{t} dt + \delta \int_{\delta}^{l_0} \frac{\omega(t)}{t^2} dt = \delta \int_{\delta}^{l_0} \frac{\omega(t)}{t(t+\delta)} dt$$

Foydalanilgan adabiyotlar.

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