

# IKKI KARRALI INTEGRALNING TATBIQLARI. IKKI KARRALI INTEGRAL YORDAMIDA YUZA VA JISM HAJMINI HISOBLASH. MASSA, O'RTA QIYMAT VA INERSIYA MOMENTI.

*Saipnazarov Jonibek Muxammadiyevich*  
*TATU Qarshi filiali Dasturiy injiniring kafedrası o'qituvchisi*  
*Usmonov Maxsud Tulqin o'g'li*  
*Toshkent axborot texnologiyalari universiteti*  
*Qarshi filiali 4-kurs talabasi*

**ANNOTATSIYA:** Ushbu maqolada matematikaning eng qiziq mavzularidan biri bo'lgan Ikki karrali integralning tatbiqlari. Ikki karrali integral yordamida yuza va jism hajmini hisoblash. Massa, o'rta qiymat va inersiya momentini topish haqida ma'lumotlar berib o'tildi va mavjud muammolarga ilmiy yondashildi. Ikki karrali integral aniq integralning ikki o'zgaruvchili(argumentli) funktsiya uchun umumlashgan holdir. Ikki karrali integral ham aniq integralning asosiy xossalari ega. Aniq integralning xossalarini takrorlashni tavsiya etamiz.

**KALIT SO'ZLAR:** Ikki karrali integralning ta'rifi, Ikki karrali integralni hisoblash, Ikki karrali integralning tatbiqlari.

**Applications of double integrals. Calculate the volume of a surface and a body using a double integral. Mass, mean and moment of inertia.**

*Saipnazarov Jonibek Muxammadiyevich*  
Teacher of Software Engineering, Karshi branch of TUIT  
*Usmonov Maxsud Tulqin o'g'li*  
Tashkent University of Information Technologies  
Karshi branch 4th year student

**ABSTRACT:** In this article, one of the most interesting topics in mathematics is the application of the double integral. Calculate the volume of a surface and a body using a double integral. Information on finding mass, mean and moment of inertia was given, and a scientific approach to the existing problems was given. A double integral is a generalized case of a definite integral for a function with two variables (arguments). The double integral also has the basic properties of a definite integral. We recommend repeating the properties of the exact integral.

**KEYWORDS:** Definition of double integral, Calculation of double integral, Applications of double integral.

## 1. Ikki karrali integralning ta'rifi.

$f(x, y)$  funktsiya biror  $D$  sohada aniqlangan bo'lsin.  $D$  sohani  $n$  ta  $D_i$  qismlarga bo'lamiz. Har bir  $D_i$  qismda  $P_i(x_i, y_i)$  bittadan nuqta tanlaymiz hamda

$$S_n = \sum_{i=1}^n f(x_i, y_i) \Delta S_i \quad (1)$$

yig'indini to'zamiz. (1) yig'indiga  $f(x, y)$  funksiya uchun  $D$  sohadagi **integral yig'indi** deyiladi.  $\lambda$  qism sohalar diametrlarining eng kattasi bo'lsin.  $\Delta S_i, D_i$  sohaning yuzi.

**Ta'rif.** (1) integral yig'indining, qismlarga bo'linish usuliga,  $P_i$  nuqtalarning tanlanishiga bog'liq bo'lmagan  $\lambda \rightarrow 0$  dagi limiti mavjud bo'lsa, bu limitga  $f(x, y)$  funksiyaning  $D$  sohadagi **ikki karrali integrali** deyiladi va

$$\iint_D f(x, y) ds$$

simvol bilan belgilanadi.

Ikki karrali integral aniq integralning ikki o'zgaruvchili(argumentli) funksiya uchun umumlashgan holidir.

Ikki karrali integral ham aniq integralning asosiy xossalariga ega. Aniq integralning xossalarini takrorlashni tavsiya etamiz.

## 2. Ikki karrali integralni hisoblash.

Ikki karrali integralni hisoblash ikkita aniq integralni ketma-ket hisoblashga keltiriladi.  $D$  soha  $y = y_1(x), y = y_2(x)$  funksiyalar graflari hamda  $x = a$  va  $x = b$  to'g'ri chiziqlar bilan chegaralangan bo'lsin, ya'ni

$$\begin{cases} a \leq x \leq b \\ y_1(x) \leq y \leq y_2(x) \end{cases}$$

tengsizliklar bilan aniqlangan bo'lsa, ikki karrali integral quyidagicha hisoblanadi:

$$\iint_D f(x, y) ds = \int_a^b \left[ \int_{y_1(x)}^{y_2(x)} f(x, y) dy \right] dx = \int_a^b dx \int_{y_1(x)}^{y_2(x)} f(x, y) dy \quad (1)$$

Oxirgi aniq integral **ichki integral** deb ataladi va uni hisoblashda  $x$  ni o'zgarmas deb, integrallash  $y$  bo'yicha olib boriladi. Ichki integralni hisoblash natijasi **tashqi integral** uchun integral osti funksiyasi bo'ladi.

$D$  soha

$$\begin{cases} c \leq y \leq d \\ x_1(y) \leq x \leq x_2(y) \end{cases}$$

tengsizliklar bilan aniqlangan bo'lsa, ikki karrali integral

$$\iint_D f(x, y) dx dy = \int_c^d \left[ \int_{x_1(y)}^{x_2(y)} f(x, y) dx \right] dy = \int_c^d dy \int_{x_1(y)}^{x_2(y)} f(x, y) dx$$

formula yordamida ikkita aniq integralni hisoblashga keltiriladi.

1-misol.  $\iint_D x \ln y dx dy$  integralni  $D$  soha:  $0 \leq x \leq 4, 1 \leq y \leq e$  to'g'ri to'rtburchak

bo'lganda hisoblang.

Yechish. (1) formulaga asosan,

$$\iint_D x \ln y dx dy = \int_0^4 x dx \int_0^e \ln y dy = \int_0^4 x dx [y \ln y - y]_1^e = \frac{x^2}{2} \Big|_0^4 = 8.$$

2-misol.  $\iint_D (x-y) dx dy$  integralni  $D: y = 2 - x^2, y = 2x - 1$ , chiziqlar bilan chegaralangan soha bo'lganda hisoblang.

Yechish. Birinchi chiziq uchi  $(0,2)$  nuqtada  $OY$  o'qiga simmetrik bo'lgan parabola. Ikkinchisi chiziq to'g'ri chiziq. Bu chiziqlarning kesishish nuqtalarini topamiz:

$$\begin{cases} y = 2 - x^2 \\ y = 2x - 1 \end{cases}$$

tenlamalar sistemasini yechib,  $A(-3;-7), B(1,1)$  nuqtalarni topamiz. (1) formulaga asosan,

$$\begin{aligned} \iint_D (x, y) dx dy &= \int_{-3}^1 dx \int_{2x-1}^{2-x^2} (x-y) dy = \\ &= \int_{-3}^1 \left[ xy - \frac{y^2}{2} \right]_{2x-1}^{2-x^2} dx = \int_{-3}^1 \left[ x \cdot (2-x^2) - \frac{(2-x^2)^2}{2} - \left[ x \cdot (2x-1) - \frac{(2x-1)^2}{2} \right] \right] dx = \\ &= \int_{-3}^1 \left( 2x - x^3 - \frac{4-4x^2+x^4}{2} - 2x^2 + x + \frac{4x^2-4x+1}{2} \right) dx = \\ &= \int_{-3}^1 \left( -\frac{1}{2}x^4 - x^3 + 2x^2 + x - \frac{3}{2} \right) dx = \\ &= \left[ -\frac{1}{10}x^5 - \frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 - \frac{3}{2}x \right]_{-3}^1 = 4\frac{4}{15} \end{aligned}$$

bo'ladi.

## 2. Ikki karrali integralning tatbiqlari.

1.  $\iint_D f(x, y) dx dy$  integralda  $f(x, y) = 1$  bo'lsa,  $\iint_D dx dy$  integral  $D$  figuraning yuzini

ifodalaydi, ya'ni

$$S = \iint_D dx dy$$

1-misol.  $x = 4y - y^2, x + y = 6$  chiziqlar bilan chegaralangan sohaning yuzini toping.

Yechish. Berilgan chiziqlarning kesishish nuqtalarini topamiz.

$x = 4y - y^2, x = 6 - y$  dan  $4y - y^2 = 6 - y, y^2 - 5y + 6 = 0,$  kesishish nuqtalari bo'ladi.  
 $y_1 = 2, y_2 = 3; x_1 = 4, x_2 = 3; A(4;2)$  va  $B(3;3)$

Shunday qilib, yuza

$$S = \iint_D dx dy = \int_2^3 dy \int_{6-y}^{4y-y^2} dx = \int_2^3 x \Big|_{6-y}^{4y-y^2} dy = \int_2^3 (4y - y^2 - 6 + y) dy = \int_2^3 (5y - y^2 - 6) dy =$$

$$= \left( \frac{5}{2} y^2 - \frac{y^3}{3} - 6y \right)_2^3 = \frac{1}{6} \text{ (kv. birlik)}$$

2. Yuqoridan  $z = f(x, y)$  sirt, quyidan  $z = 0$  tekislik, yon tomondan to'g'ri silindrik sirt bilan hamda  $XOY$  tekislikda  $D$  sohani hosil qiladigan **silindrik jismning xajmi**

$$V = \iint_D f(x, y) dx dy \text{ integral bilan xisoblanadi.}$$

2-misol.  $y = 1 + x^2$ ,  $z = 3x$ ,  $y = 5$ ,  $z = 0$  sirtlar bilan chegaralangan I oktantadagi jismning hajmini hisoblang.

Yechish. Hajmi hisoblanishi kerak bo'lgan jism yuqoridan  $z = 3x$  tekislik, yondan  $y = 1 + x^2$  parabolik silindr,  $y = 5$  tekislik bilan chegaralangan. Shunday kilib

$$V = \iint_D 3x dx dy = 3 \int_0^2 x dx \int_{1+x^2}^5 dy = 3 \int_0^2 x [5 - (1 + x^2)] dx = 3 \int_0^2 (4x - x^3) dx = 3 \left( 4 \frac{x^2}{2} - \frac{x^4}{4} \right)_0^2 = 3 \left( 2 \cdot 2^2 - \frac{2^4}{4} \right) = 24 - 12 = 12 \text{ куб.бур.}$$

Plastinka har bir nuqtasidagi zichlik funksiyasi  $\gamma(x, y)$  bo'lsa, uning massasi

$$m = \iint_D \gamma(x, y) dx dy$$

integral bilan hisoblanadi.

Plastinkaning  $OX$  va  $OY$  o'qlarga nisbatan **statik momentlari**.

$$M_x = \iint_D y \gamma(x, y) dx dy, \quad M_y = \iint_D x \gamma(x, y) dx dy$$

formulalar bilan hisoblanadi.

Plastinka birjinsli, ya'ni  $\gamma = \cos nt$  bo'lganda uning **og'irlik markazining koordinatalari**

$$\bar{x}_c = \frac{M_y}{S} = \frac{\iint_D x dx dy}{S}, \quad \bar{y}_c = \frac{M_x}{S} = \frac{\iint_D y dx dy}{S}$$

formulalar yordamida topiladi, bu yerda  $S$ ,  $D$  sohaning yuzi.

Plastinkaning  $OX$  va  $OY$  o'qlariga nisbatan **inertsia momentlari**

$$J_x = \iint_D y^2 \gamma(x, y) dx dy, \quad J_y = \iint_D x^2 \gamma(x, y) dx dy$$

formulalar bilan, koordinatlar boshiga nisbatan inertsia momenti

$$J_0 = \iint_D (x^2 + y^2) \gamma(x, y) dx dy = J_x + J_y$$

formula bilan aniqlanadi. Yuqoridagi formulalarda  $\gamma(x, y) = 1$  deb tekis figuralarning geometrik inertsia momentlarini topish formulalarini olamiz.

3-misol.  $y^2 = 4x + 4$ ,  $y^2 = -2x + 4$  chiziqlar bilan chegaralangan figuraning og'irlik markazining koordinatlarini toping.

Yechish. Chiziqlar  $OX$  o'qiga nisbatan simmetrik bo'lganligi uchun  $\bar{y}_c = 0$   $\bar{x}_c$  ni topamiz:

$$S = \iint_D dx dy = 2 \int_0^2 dy \int_{\frac{y^2-4}{4}}^{\frac{4-y^2}{2}} dx = 2 \int_0^2 \left( \frac{4-y^2}{2} - \frac{y^2-4}{4} \right) dy = 2 \int_0^2 \left( 3 - \frac{3y^2}{4} \right) dy =$$

$$= 6 \left[ y - \frac{y^3}{12} \right]_0^2 = 6 \left( 2 - \frac{8}{12} \right) = 8$$

$$\bar{x}_c = \frac{1}{8} \iint_L x dx dy = \frac{1}{8} 2 \int_0^2 dy \int_{\frac{y^2-4}{4}}^{\frac{4-y^2}{2}} x dx dy = \frac{1}{8} \int_0^2 \left[ \frac{4-y^2}{4} - \frac{(y^2-4)}{16} \right] dy = \frac{1}{8} \int_0^2 \left( 3 - \frac{3}{2} y^2 + \frac{3}{16} y^4 \right) dy =$$

$$= \frac{1}{8} \left( 3y - \frac{y^3}{2} + \frac{3y^5}{80} \right)_0^2 = \frac{2}{5}. \quad \text{Demak } C\left(\frac{2}{5}; 0\right).$$

#### Adabiyotlar ro'yxati:

1. Claudio Canuto, Anita Tabacco "Mathematical Analysis", Italy, Springer, I-part, 2008, II-part, 2010.
2. W. WL.Chen "Linear algebra", London, Chapter 1-12, 1983, 2008.
3. W.WL.Chen "Introduction to Fourier Series", London, Chapter 1-8, 2004, 2013.
4. W.WL.Chen "Fundamentals of Analysis", London, Chapter 1-10, 1983, 2008.
5. Soatov Yo U. Oliy matematika. T., O'qituvhi, 1995. 1- 5 qismlar.
6. Azlarov T., Mansurov X. Matematik analiz, - Toishkent, O'qituvhi, 1-qism, 1989.
7. Бугров Я.С., Никольский С.М. Дифференциальные уравнения. Кратные интегралы. Рады. Функции комплексного переменного. - Наука, 1997.
8. V.Ye.Shneyder, A.I.Slutskiy, A.S.Shumov. Qisqaha oliy matematika kursi. T., 1985., 2-qism.
9. Беклемишев. Д.В. Курс аналитической геометрии и линейной алгебры. -М.: Наука, 1984.
10. Бугров Я.С., Никольский С.М. Дифференциальное и интегральное исчисление. М.: Наука, 1983.