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MOMENTLAR YAQINLASHISH MASALALARI

Annotatsiya: Maqolada $\Gamma_k\{\xi\}$ ni o‘sish tartibiga qarab belgilangan tengsizliklar asosida λ_k ni baholash mumkin ekan. Biz dastlab ikkita lemmani isbotlaymiz, so‘ng ulardan foydalanibX $E\xi = 0$ va $D\xi = 1$ bo‘lganda ξ tasodifiy miqdor uchun (S_γ) tengsizlik $H = \frac{1}{2}$ bo‘lganda bahosini aniqlaymiz.

Kalit so‘zlar: ξ tasodifiy miqdor, $M_k - k$ – tartibli absalyut moment, $\Gamma_k\{\xi\}$ ξ tasodifiy miqdorni k −tartibli semiinvarianti,

$F_\xi(x)$ ixtiyoriy ξ tasodifiy miqdorning taqsimot funksiyasi bo‘lsin, $\Phi(x) - N(0;1)$ – normal taqsimot. Quyidagi belgilashlarni kiritamiz:

$$Q = sup\left|F_\xi(x) - \Phi(x)\right|,$$

$M_k - k$ – tartibli absalyut momentga ega bo‘lgan taqsimot funksiyalar to‘plami.

Ma’lumki $0 < Q \leq \frac{1}{\sqrt{e}}$ va $F_\xi(x) \epsilon M_k$ sharti ostida quyidagi baho topilgan. Barcha x lar uchun

$$\left|F_\xi(x) - \Phi(x)\right| \leq \frac{c_k Q \left(log\frac{1}{Q}\right)^{\frac{k}{2}} + \lambda_k}{1 + |x|^k}, \quad (1)$$

Bu yerda c_k – faqat k ga bog’liq o‘zgarmas,

$$\lambda_k = \left|\int_{-\infty}^{\infty} |x|^k dF_\xi(x) - \int_{-\infty}^{\infty} |x|^k d\Phi_\xi(x)\right|. \quad (2)$$

(1) bahoni amaliy yoki nazariy masalalarni tahlil qilishda ishlatish uchun (2) ni baholash kerak bo‘ladi. Shu nuqtai nazardan biz ba’zi shartlar asosida uni baholash masalalarini xal qilamiz.

$E\xi = 0$ va $D\xi = 1$ bo‘lgan ξ tasodifiy miqdorni k −tartibli semiinvariantini $\Gamma_k\{\xi\}$ kabi belgilaymiz.

$\Gamma_k\{\xi\}$ ni o‘sish tartibiga qarab belgilangan tengsizliklar asosida λ_k ni baholash mumkin ekan. Biz dastlab ikkita yordamchi teoremani isbotlaymiz, so‘ng ulardan foydalanib (2) ni bahosini aniqlaymiz

1 – lemma E$\xi = 0$ va $D\xi = 1$ bo‘lgan ξ tasodifiy miqdorning k – tartibli semiinvarianti quyidagi tengsizlikni qanoatlantirsin:

$$|\Gamma_k\{\xi\}| \le \frac{H(k-2)!}{\Delta^{k-2}}, k = 3,4,5,\dots, s+2 \qquad (S)$$

bu yerda s juft va s$< 2\Delta^2$ tengsizlikni qanoatlantiradi, $H > 0$ va $\Delta > 0$ o‘zgarmas miqdor. U holda quyidagi tengsizlik o‘rinli bo‘ladi:

$$|m_k| \le \beta_k \le \begin{cases} \frac{k!\, e^H}{\sqrt{e}\Delta} + \frac{k!\, H^{\frac{k}{2}}}{\left(\frac{k}{2}\right)!}, agar\ k = 2n; \\ \frac{k!\, e^H}{\sqrt{e}\Delta}, agar\ k = 2n+1. \end{cases} \qquad (M)$$

bu yerda $m_k = E\xi^k$, $\beta_k = E|\xi|^k$ $, = 1,2,3,\dots$

Isbot: Momentlarni semiinvariantlar bilan bog’lanish formulasidan foydalanamiz:

$$m_k = k! \sum_{j=1}^{\left[\frac{k}{2}\right]} \frac{1}{j!} \sum_{\substack{k_1+k_2+\cdots+k_j=k \\ k_j=2,3,\dots,S \\ 2\le S\le 2j}} \frac{\Gamma_{k_1}\{\xi\}\cdot\Gamma_{k_2}\{\xi\}\cdot\ldots\cdot\Gamma_{k_j}\{\xi\}}{k_1!\,k_2!\,\ldots\,k_j!}$$

Bu yerda $[e] - e$ ni butun qismi.

(S) dan foydalanib $|m_k|$ ni baholaymiz.

$$|m_k| \le k! \sum_{j=1}^{\left[\frac{k}{2}\right]} \frac{1}{j!} \sum_{2\le s\le 2j} \prod_{i=2}^{s} \frac{1}{i(i-1)} \cdot \frac{H^j}{\Delta^{k-2j}} \le \frac{k!}{\sqrt{e}} \sum_{j=1}^{\left[\frac{k}{2}\right]} \frac{1}{j!} \cdot \frac{H^j}{\Delta^{k-2j}}. \qquad (3)$$

Faraz qilamiz $k = 2n$ bo‘lsin. U holda (3) dan

$$|m_{2n}| \leq \frac{(2n)!}{\sqrt{e}} \cdot \left(\frac{1}{1!} \cdot \frac{H}{\Delta^{2n-2}} + \frac{1}{2!} \cdot \frac{H^2}{\Delta^{2n-4}} + \cdots + \frac{1}{(n-1)!} \cdot \frac{H^{n-1}}{\Delta^2}\right) + \frac{(2n)!}{\sqrt{e}} \cdot \frac{H^n}{n!}$$

$$\leq \frac{(2n)!}{\sqrt{e}\Delta^2}\left(\frac{H}{1!} + \frac{H^2}{2!} + \cdots + \frac{H^n}{n!} + \cdots\right) + \frac{(2n)!\,H^n}{n!}$$

$$\leq \frac{(2n)!\,e^H}{\sqrt{e}\,\sqrt{\Delta}} + \frac{(2n)!\,H^n}{n!}$$

$k = 2n + 1$ bo‘lganda esa

$$|m_{2n+1}| \leq \frac{(2n+1)!\,e^H}{\sqrt{e}\Delta}$$

Ham shu kabi hosil bo‘ladi.

$m_{2n} = \beta_{2n}$ va $\beta_{2n+2} = \begin{cases} m_{2n+1}, & x > 0 \\ -m_{2n+1}, & x < 0 \end{cases}$ bo‘lgani uchun tengsizlik β_k uchun ham to‘g’ri bo‘ladi.

1- lemma isbotlandi.

<u>*2 – lemma:*</u> Agar $E\xi = 0$ va $D\xi = 1$ bo‘lgan ξ tasodifiy miqdorni k – tartibli semiinvarianti uchun quyidagi tengsizlik o‘rinli bo‘lsin:

$$\Gamma_k\{\xi\} \leq \frac{H(k!)^{1+\gamma}}{\Delta^{k-2}}, k = 3,4,5, \ldots \qquad (S_\gamma)$$

Bu yerda $H > 0$, $\Delta > 0, \gamma > 0$.

U holda

$$|m_k| \leq \begin{cases} \frac{6 \cdot 2^{2\gamma} e^H}{\sqrt{e}} \left(\frac{\sqrt{2}}{6}\right)^{\frac{2\gamma}{1+2\gamma}} \frac{k!}{\Delta^{\frac{1}{1+2\gamma}}} + \frac{k!\,H^{\frac{k}{2}}}{\left(\frac{k}{2}\right)!}, agar\ k = 2n; \\ \frac{6 \cdot 2^{2\gamma} e^H}{\sqrt{e}} \left(\frac{\sqrt{2}}{6}\right)^{\frac{2\gamma}{1+2\gamma}} \frac{k!}{\Delta^{\frac{1}{1+2\gamma}}}, agar\ k = 2n+1. \end{cases}$$

<u>*Isbot:*</u> Quyidagi tengsizliklardan foydalanamiz

$$\frac{(k!)^{1+\gamma}}{\Delta^{k-2}} \leq (k-2)!\left(\frac{6(s+2)^\gamma}{\Delta}\right)^{k-2}, \quad k = 3,4, \ldots, s+2, s \geq 4,$$

$$2 \leq \sqrt{s} < \left(\frac{\sqrt{2}}{6}\Delta\right)^{\frac{1}{1+2\gamma}}$$

Bulardan foydalanib (S_γ) ni baholaymiz

$$|\Gamma_k\{\xi\}| \leq H(k-2)!\left(\frac{6(S+2)^\gamma}{\Delta}\right)^{k-2} = \frac{H(k-2)!}{\Delta_S^{k-2}},$$

Bu yerda $\Delta_S = \frac{\Delta}{6(S+2)^\gamma}$.

U holda (M) ni ko‘rinishi quyidagicha bo‘ladi.

$$|m_k| \leq \beta_k \leq \begin{cases} \frac{k!\,e^H}{\sqrt{e}\Delta_S} + \frac{k!\,H^{\frac{k}{2}}}{\left(\frac{k}{2}\right)!}, agar\ k = 2n; \\ \frac{k!\,e^H}{\sqrt{e}\Delta_S}, agar\ k = 2n+1. \end{cases} \qquad (M_S)$$

va

$$(s+2)^\gamma < 2^{2\gamma}\left(\frac{\sqrt{2}}{6}\Delta\right)^{\frac{2\gamma}{1+2\gamma}}$$

yuqoridagilardan foydalanib (M_S) ni baholaymiz. Agar $k = 2n$ bo‘lsa

$$|m_{2n}| \leq \frac{(2n)!\,e^H}{\sqrt{e}} \cdot \frac{6(S+2)^\gamma}{\Delta} + \frac{(2n)!\,H^n}{n!}$$

$$\leq \frac{6 \cdot 2^{2\gamma} e^H}{\sqrt{e}}\left(\frac{\sqrt{2}}{6}\Delta\right)^{\frac{2\gamma}{1+2\gamma}} \frac{(2n)!}{\Delta^{\frac{1}{1+2\gamma}}} + \frac{(2n)!\,H^n}{n!};$$

$k = 2n+1$ da esa

$$|m_{2n}| \leq \frac{(2n+1)!\,e^H}{\sqrt{e}} \cdot \frac{6(S+2)^\gamma}{\Delta} \leq \frac{6 \cdot 2^{2\gamma} e^H}{\sqrt{e}}\left(\frac{\sqrt{2}}{6}\Delta\right)^{\frac{2\gamma}{1+2\gamma}} \frac{(2n+1)!}{\Delta^{\frac{1}{1+2\gamma}}}$$

2 – lemma isbot bo‘ldi.

Teorema $E\xi = 0$ va $D\xi = 1$ bo‘lganda ξ tasodifiy miqdor uchun (S_γ) tengsizlik $H = \frac{1}{2}$ bo‘lganda o‘rinli bo‘lsin. U holda

$$|\lambda_k| \leq 6 \cdot 2^{2\gamma}\left(\frac{\sqrt{2}}{6}\right)^{\frac{2\gamma}{1+2\gamma}} \frac{k!}{\Delta^{\frac{1}{1+2\gamma}}}. \qquad (M_\lambda)$$

Isbot: $\Phi(x) - N(0,1)$ – momentlari uchun quyidagilar ma’lum|

$$m_{2n} = \beta_{2n}^{n} = \frac{(2n)!}{2^n n!}, m_{2n+1}^{n} = 0.$$

u holda

$$m_{2n+1} \le |m_{2n+1}| \le \beta_{2n+1} \quad ,$$

$$|\beta_{2n} - \beta_{2n}^{n}| = |m_{2n} - m_{2n}^{n}| \quad ,$$

$$|\beta_{2n+1} - \beta_{2n+1}^{n}| = \left|\beta_{2n+1} - |m_{2n+1}^{n}|\right| = |\beta_{2n+1}| \; .$$

munosabatlardan va 2 – lemmadan (M_λ) kelib chiqadi.

Teorema isbot bo‘ldi.

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