

## **TWO BOUNDARIES OF NATURAL COLLISIONS - ABSOLUTE ELASTIC AND INELASTIC COLLISIONS.**

### **Abstract:**

The following article is aimed at discussing the methods used in academic lyceums for teaching elastic and inelastic collisions.

### **Key Words:**

Absolute inelastic collision; absolute elastic collision; coefficient of restitution; loss of energy.

When ordinary objects collide, this results in conversion of some portion of the energy into internal energy. Depending on the value of the coefficient of restitution, mechanical energy can be lost. Collision with the coefficient of recovery  $a = 0$  are called as absolutely inelastic collisions. Such collision results in the loss of the most mechanical energy. Collision with a coefficient of restitution  $a = 1$  are called absolutely elastic collisions. In the cases of such collision, mechanical energy is lost to a minimum, ie not lost at all. Normally, the coefficients of restitution of energy in cases of collisions between two bodies lie in the range of absolute inelasticity and absolute elasticity, so to say in the range  $0 < a < 1$ . We will discuss such collisions in more detail.

### **Absolute inelastic collision:**

*Absolute inelastic collision can be defined as a collision in which the momentum of the system is conserved and the colliding bodies move at the same speed as a result.*

In a case of absolute inelastic collision, the colliding bodies move if they were one connected mass. An example can be a person jumping into the moving car or a projectile stuck in a sandy platform (Figure-1).

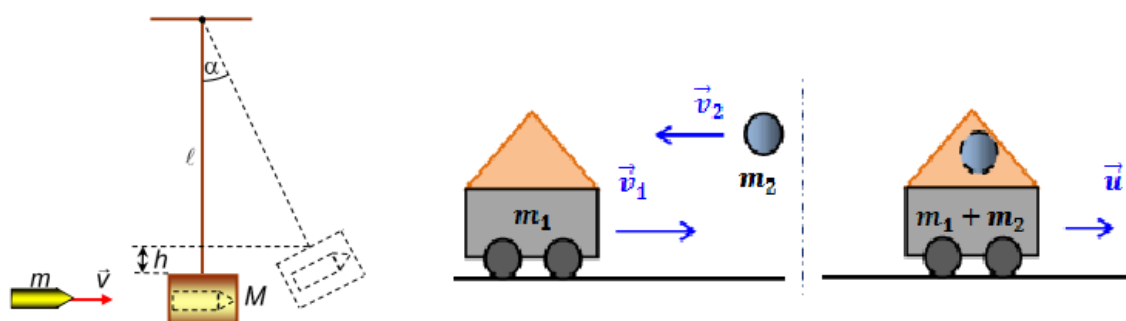


Figure-1

In case of absolute inelastic collision, the maximum possible kinetic energy has been lost, that is, it is converted into internal energy (heating). In no case of collisions happening in nature, more kinetic energy is lost as in cases of absolute inelastic collision. In such cases, the collided bodies start moving as if they were one (Figure-1).

The formula of conservation of momentum for inelastic collision is:

$$\vec{p}_1 + \vec{p}_2 = \vec{p}' \quad \text{yoki} \quad m_1 \vec{g}_1 + m_2 \vec{g}_2 = (m_1 + m_2) \vec{g}'$$

In many cases, we discuss collisions that happen in one line. Therefore, in the above formula for an absolute inelastic collision, we considered that bodies moved in a straight line, on the axis Ox. To avoid confusion, we can enumerate the following situations:

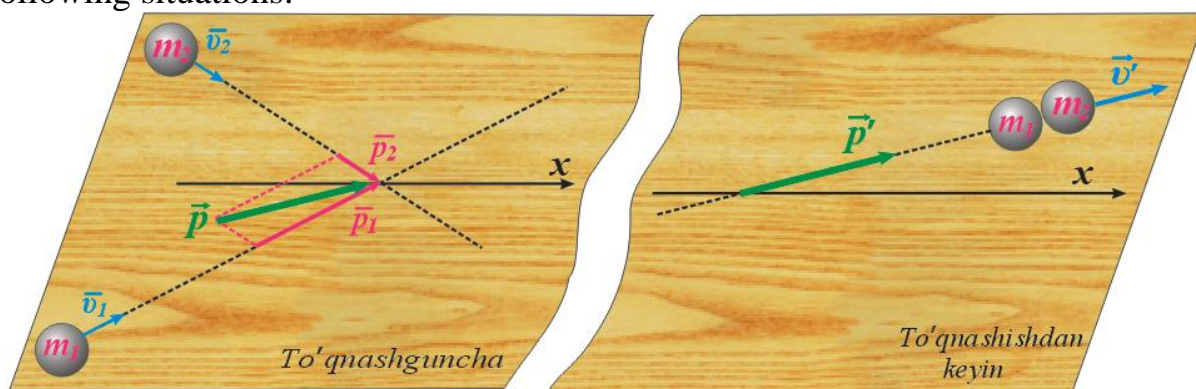


Figure-1

1.1) Let one object chase another and hit it absolutely inelastically. The velocity of bodies after the collision can be determined using the law of conservation of momentum.

$$m_1 g_1 + m_2 g_2 = (m_1 + m_2) g', \quad \rightarrow \quad g' = \frac{m_1 g_1 + m_2 g_2}{m_1 + m_2}$$

This means that if a balloon moving at velocity  $g_1$  follows the trajectory of a balloon moving at velocity  $g_2$  and hits it with absolute inelasticity, the velocity after the collision will be as follows (Figure-3):

$$g' = \frac{m_1 g_1 + m_2 g_2}{m_1 + m_2}$$

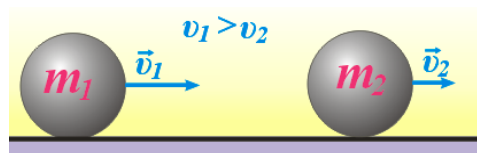


Figure-3

The above formula can also be derived by substituting  $\alpha=0$  for the real multiplication formula provided by the coefficient of restitution.

$$\begin{cases} \mathcal{G}'_1 = \frac{m_1 \mathcal{G}_1 + m_2 \mathcal{G}_2 - 0 \cdot m_2 (\mathcal{G}_1 - \mathcal{G}_2)}{m_1 + m_2} = \frac{m_1 \mathcal{G}_1 + m_2 \mathcal{G}_2}{m_1 + m_2} \\ \mathcal{G}'_2 = \frac{m_1 \mathcal{G}_1 + m_2 \mathcal{G}_2 + 0 \cdot m_1 (\mathcal{G}_1 - \mathcal{G}_2)}{m_1 + m_2} = \frac{m_1 \mathcal{G}_1 + m_2 \mathcal{G}_2}{m_1 + m_2} \end{cases} ; \Rightarrow \mathcal{G}'_1 = \mathcal{G}'_2 = \mathcal{G}' = \frac{m_1 \mathcal{G}_1 + m_2 \mathcal{G}_2}{m_1 + m_2}$$

1.2) Let us assume that the masses defined in the above formula are equal:  $m_1=m_2$ , the velocity after the collision would be:

$$\mathcal{G}' = \frac{\mathcal{G}_1 + \mathcal{G}_2}{2}$$

2.1) Let one object strike another object moving in the opposite direction. The collision is absolute inelastic. The velocity after the collision is determined by using the the formula of conservation of momentum:

$$m_1 \mathcal{G}_1 + m_2 \mathcal{G}_2 = (m_1 + m_2) \mathcal{G}' , \rightarrow \mathcal{G}' = \frac{m_1 \mathcal{G}_1 - m_2 \mathcal{G}_2}{m_1 + m_2}$$

This means that if a ball moving at velocity  $\mathcal{G}_1$  strikes a ball moving at velocity  $\mathcal{G}_2$  with absolute inelasticity, the velocity after the collision will be as follows (Figure-4):

$$\mathcal{G}' = \frac{m_1 \mathcal{G}_1 - m_2 \mathcal{G}_2}{m_1 + m_2}$$

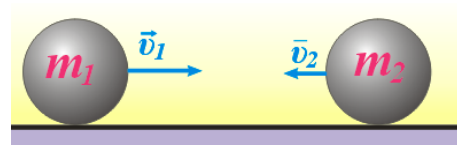


Figure-4

The above formula can also be derived by substituting  $\alpha=0$  for the real multiplication formula provided by the coefficient of restitution.

$$\begin{cases} \mathcal{G}'_1 = \frac{m_1 \mathcal{G}_1 - m_2 \mathcal{G}_2 - 0 \cdot m_2 (\mathcal{G}_1 + \mathcal{G}_2)}{m_1 + m_2} = \frac{m_1 \mathcal{G}_1 - m_2 \mathcal{G}_2}{m_1 + m_2} \\ \mathcal{G}'_2 = \frac{m_1 \mathcal{G}_1 - m_2 \mathcal{G}_2 + 0 \cdot m_1 (\mathcal{G}_1 + \mathcal{G}_2)}{m_1 + m_2} = \frac{m_1 \mathcal{G}_1 - m_2 \mathcal{G}_2}{m_1 + m_2} \end{cases} ; \Rightarrow \mathcal{G}'_1 = \mathcal{G}'_2 = \mathcal{G}' = \frac{m_1 \mathcal{G}_1 - m_2 \mathcal{G}_2}{m_1 + m_2}$$

2.2) Let us assume that the masses defined in the above formula are equal:  $m_1=m_2$ , the velocity after the collision would be:

$$\mathcal{G}' = \frac{\mathcal{G}_1 - \mathcal{G}_2}{2}$$

3.1) Let one object strike another object that is staying still. The momentum of the first ball is converted into the momentum of both balls after the collision.

$$m_1 \mathcal{G}_1 = (m_1 + m_2) \mathcal{G}' , \rightarrow \mathcal{G}' = \frac{m_1 \mathcal{G}_1}{m_1 + m_2}$$

This means that if a ball moving at velocity  $g_1$  strikes a ball staying still with absolute inelasticity, the velocity after the collision will be as follows (Figure-5):

$$g' = \frac{m_1}{m_1 + m_2} g_1$$

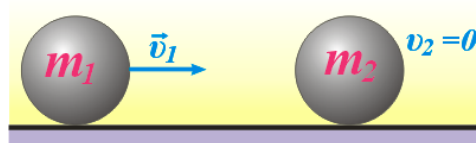


Figure-5

The above formula can also be derived by substituting  $\alpha=0$  for the real multiplication formula given by the recovery coefficient in the previous topic.

$$\begin{cases} g'_1 = \frac{m_1 - 0 \cdot m_2}{m_1 + m_2} g_1 = \frac{m_1 g_1}{m_1 + m_2} \\ g'_2 = \frac{(1+0)m_1}{m_1 + m_2} g_1 = \frac{m_1 g_1}{m_1 + m_2} \end{cases} ; \Rightarrow g'_1 = g'_2 = g' = \frac{m_1 g_1}{m_1 + m_2}$$

3.2) Let us assume that the masses defined in the above formula are equal:  $m_1=m_2$ , the velocity after the collision would be:

$$g' = \frac{g_1}{2}$$

From the above formula it follows that if there are n-number of balls in a line, one of which moves with velocity and hits the remaining balls absolutely inelastically, this results in n-balls moving with  $g' = g/n$  velocity.

If an object collides absolutely inelastically with another moving object with a very large mass, the velocity after the collision is equal to the velocity of a heavy object. This is because a light object does not significantly change the speed or momentum of a heavy object. If a heavy object is stationary, the velocity after the collision is zero. An example of this is when an object thrown into the sand from a height gets stuck in the sand.

As mentioned above, for absolutely inelastic collisions, the recovery coefficients are  $\alpha=0$ . As a result of an absolutely inelastic collision, most of the kinetic energy is converted into internal energy. Therefore, the absolute inelastic collision can be called the lower energy limit at which kinetic energy is lost the most.

### **Absolute elastic collision:**

An elastic collision is a collision in which there is no net loss in kinetic energy in the system as a result of the collision. Both momentum and kinetic energy are conserved quantities in elastic collision (Figure-6).

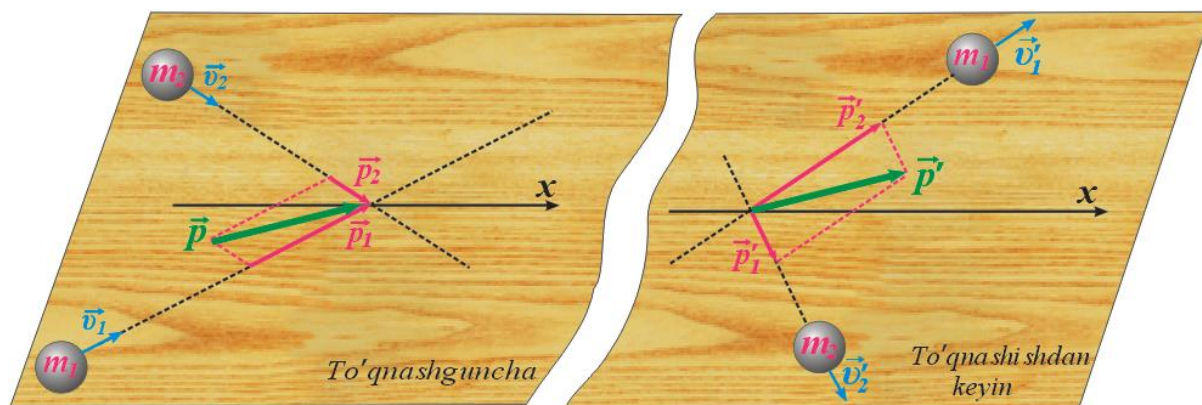


Figure-6

$$\left\{ \begin{array}{l} \vec{p}_1 + \vec{p}_2 = \vec{p}'_1 + \vec{p}'_2 \\ E_1 + E_2 = E'_1 + E'_2 \end{array} \right. \quad \text{yoki} \quad \left\{ \begin{array}{l} m_1 \vec{g}_1 + m_2 \vec{g}_2 = m_1 \vec{g}'_1 + m_2 \vec{g}'_2 \\ \frac{m_1 g_1^2}{2} + \frac{m_2 g_2^2}{2} = \frac{m_1 g_1'^2}{2} + \frac{m_2 g_2'^2}{2} \end{array} \right.$$

In many cases, we discuss collisions that happen in one line. Therefore, in the above formula for an absolute inelastic collision, we considered that bodies moved in a straight line, on the axis Ox. To avoid confusion, we can enumerate the following situations:

1.1) Let one body chase another and hit it absolutely elastically. The velocity after a collision is determined by using the law of conservation of momentum and energy. The collision is central. Therefore, the velocity vectors are directed along the line connecting the centers of the objects. Because the velocity vectors are in a straight line, it is possible to switch from vector view to scalar view. By using the above formula:

$$\left\{ \begin{array}{l} m_1 g_1 + m_2 g_2 = m_1 g'_1 + m_2 g'_2 \\ \frac{m_1 g_1^2}{2} + \frac{m_2 g_2^2}{2} = \frac{m_1 g_1'^2}{2} + \frac{m_2 g_2'^2}{2} \end{array} \right.$$

we get:

$$\left\{ \begin{array}{l} m_1 (g_1 - g'_1) = m_2 (g'_2 - g_2) \\ m_1 (g_1^2 - g_1'^2) = m_2 (g_2'^2 - g_2^2) \end{array} \right.$$

From this formula, we get:  $m_1 (g_1 - g'_1)(g_1 + g'_1) = m_2 (g'_2 - g_2)(g'_2 + g_2)$ , which further can be shortened as  $g_1 + g'_1 = g_2 + g'_2$ . Hence, we find that the velocity of the second ball as  $g'_2 = g_1 + g'_1 - g_2$ , which, then, is incorporated into above formula. Thus, we get:

$$m_1 (g_1 - g'_1) = m_2 (g_1 + g'_1 - g_2 - g_2), \rightarrow m_1 g_1 - m_1 g'_1 = m_2 g_1 + m_2 g'_1 - 2m_2 g_2, \rightarrow (m_1 - m_2) g_1 + 2m_2 g_2 = (m_1 + m_2) g'_1$$

The formula of velocity can take the form of:

$$g'_1 = \frac{(m_1 - m_2) g_1 + 2m_2 g_2}{m_1 + m_2}$$

If we incorporate this, into  $\mathcal{G}'_2 = \mathcal{G}_1 + \mathcal{G}'_1 - \mathcal{G}_2$ , we get:

$$\begin{aligned}\mathcal{G}'_2 &= \mathcal{G}_1 + \frac{(m_1 - m_2)\mathcal{G}_1 + 2m_2\mathcal{G}_2}{m_1 + m_2} - \mathcal{G}_2 = \frac{m_1\mathcal{G}_1 + m_2\mathcal{G}_1 + m_1\mathcal{G}_1 - m_2\mathcal{G}_1 + 2m_2\mathcal{G}_2 - m_1\mathcal{G}_2 - m_2\mathcal{G}_2}{m_1 + m_2} = \\ &= \frac{(m_2 - m_1)\mathcal{G}_2 + 2m_1\mathcal{G}_1}{m_1 + m_2}\end{aligned}$$

Thus, the above formula can also be derived by substituting  $\alpha=1$  for the real multiplication formula provided by coefficient of restitution.

$$\begin{cases} \mathcal{G}'_1 = \frac{m_1\mathcal{G}_1 + m_2\mathcal{G}_2 - 1 \cdot m_2(\mathcal{G}_1 - \mathcal{G}_2)}{m_1 + m_2} = \frac{m_1\mathcal{G}_1 + m_2\mathcal{G}_2 - m_2\mathcal{G}_1 + m_2\mathcal{G}_2}{m_1 + m_2} = \frac{(m_1 - m_2)\mathcal{G}_1 + 2m_2\mathcal{G}_2}{m_1 + m_2} \\ \mathcal{G}'_2 = \frac{m_1\mathcal{G}_1 + m_2\mathcal{G}_2 + 1 \cdot m_1(\mathcal{G}_1 - \mathcal{G}_2)}{m_1 + m_2} = \frac{m_1\mathcal{G}_1 + m_2\mathcal{G}_2 + m_1\mathcal{G}_1 - m_1\mathcal{G}_2}{m_1 + m_2} = \frac{(m_2 - m_1)\mathcal{G}_2 + 2m_1\mathcal{G}_1}{m_1 + m_2} \end{cases}$$

1.2) Let us assume that the masses defined in the above formula are equal:  $m_1=m_2$ , the velocity after the collision would be:

$$\begin{cases} \mathcal{G}'_1 = \mathcal{G}_2 \\ \mathcal{G}'_2 = \mathcal{G}_1 \end{cases}$$

2.1) Let one object strike the other object moving towards the first object with absolute elasticity.

If a ball traveling at velocity  $\mathcal{G}_1$  strikes a ball traveling at velocity  $\mathcal{G}_2$  with absolute elasticity, the velocities after the collision are as follows (Figure-8):

$$\begin{cases} \mathcal{G}'_1 = \frac{(m_1 - m_2)\mathcal{G}_1 - 2m_2\mathcal{G}_2}{m_1 + m_2} \\ \mathcal{G}'_2 = \frac{(m_1 - m_2)\mathcal{G}_2 + 2m_1\mathcal{G}_1}{m_1 + m_2} \end{cases}$$

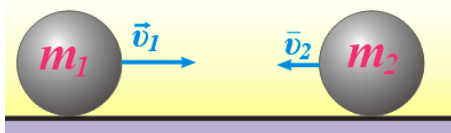


Figure-8

The above formula can also be derived by substituting  $\alpha=1$  for the real multiplication formula provided by the coefficient of restitution.

$$\begin{cases} \mathcal{G}'_1 = \frac{m_1\mathcal{G}_1 - m_2\mathcal{G}_2 - 1 \cdot m_2(\mathcal{G}_1 + \mathcal{G}_2)}{m_1 + m_2} = \frac{m_1\mathcal{G}_1 - m_2\mathcal{G}_2 - m_2\mathcal{G}_1 - m_2\mathcal{G}_2}{m_1 + m_2} = \frac{(m_1 - m_2)\mathcal{G}_1 - 2m_2\mathcal{G}_2}{m_1 + m_2} \\ \mathcal{G}'_2 = \frac{m_1\mathcal{G}_1 - m_2\mathcal{G}_2 + 1 \cdot m_1(\mathcal{G}_1 + \mathcal{G}_2)}{m_1 + m_2} = \frac{m_1\mathcal{G}_1 - m_2\mathcal{G}_2 + m_1\mathcal{G}_1 + m_1\mathcal{G}_2}{m_1 + m_2} = \frac{(m_1 - m_2)\mathcal{G}_2 + 2m_1\mathcal{G}_1}{m_1 + m_2} \end{cases}$$

2.2) Let us assume that the masses defined in the above formula are equal:  $m_1=m_2$ , the velocity after the collision would be:

$$\begin{cases} \mathcal{G}'_1 = -\mathcal{G}_2 \\ \mathcal{G}'_2 = \mathcal{G}_1 \end{cases}$$



3.1) Let one object hit another object staying still with absolute elasticity. If an object moving at velocity  $\mathcal{V}_1$  hits an object staying still with absolute elasticity, the velocities after the collision will be as follows (Figure-9):

$$\begin{cases} \mathcal{V}'_1 = \frac{m_1 - m_2}{m_1 + m_2} \mathcal{V}_1 \\ \mathcal{V}'_2 = \frac{2m_1}{m_1 + m_2} \mathcal{V}_1 \end{cases}$$



Figure-9

The above formula can also be derived by substituting  $\alpha=1$  for the real multiplication formula provided by the coefficient of restitution.

$$\begin{cases} \mathcal{V}'_1 = \frac{m_1 - \alpha \cdot m_2}{m_1 + m_2} \mathcal{V}_1 = \frac{m_1 - 1 \cdot m_2}{m_1 + m_2} \mathcal{V}_1 = \frac{m_1 - m_2}{m_1 + m_2} \mathcal{V}_1 \\ \mathcal{V}'_2 = \frac{(1 + \alpha)m_1}{m_1 + m_2} \mathcal{V}_1 = \frac{(1 + 1)m_1}{m_1 + m_2} \mathcal{V}_1 = \frac{2m_1}{m_1 + m_2} \mathcal{V}_1 \end{cases}$$

3.2) Let us assume that the masses defined in the above formula are equal:  $m_1=m_2$ , the velocity after the collision would be:

$$\begin{cases} \vec{\mathcal{V}}'_1 = 0 \\ \vec{\mathcal{V}}'_2 = \mathcal{V}_1 \end{cases}$$

From the above formula it follows that there are n-number of balls staying in one line, among which one ball moves with velocity  $\mathcal{V}$  and hits remaining balls with absolute elasticity, the last standing ball starts moving with the velocity  $\mathcal{V}' = \mathcal{V}$ . This is well demonstrated at Figure-10.



a)



b)

Figure-10

As mentioned above, for absolute elastic colliding bodies, the coefficient of restitution is  $\alpha=1$ . As a result of absolute elastic impact, the kinetic energy is converted into internal energy. Therefore, an absolute elastic collision is a collision, where kinetic energy is lost the least. In nature, there are no collision resulting in the loss of less kinetic energy, as it would contradict the law of conservation of energy.

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