ANIQ INTEGRALLARNING BA'ZI TADBIQLARI ORQALI MATEMATIKA FANINI KASBGA YO'NALTIRIB O'QITISH KO'NIKMASINI SHAKLLANTIRISH

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Annotatsiya: Ushbu maqolada aniq integrallar tadbiqlariga doir ba'zi aylanish jismlarini hajmini hisoblashga oid misollarning ishlanish usullari ko'rib chiqilgan va oliy ta'lim muassasalari talabalariga o'rgatishning qulay metodikasi tahlil qilingan.

Kalit so'zlar: integrallar, aniq integrallar, aylanma jism hajmi, aniq integral tadbiqlari.

DEVELOPING THE PROFESSION OF TEACHING MATHEMATICS TO PROFESSIONALLY THROUGH SOME APPLICATIONS OF EXACT INTEGRALS

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Abstract: This article discusses the development of examples of the calculation of the volume of some rotating bodies on the application of exact integrals and analyzes a convenient method of teaching students of higher education institutions.

Keywords: integrals, exact integrals, rotating body volume, exact integral applications.

KIRISH

O'zbekiston Respublikasi Prezidentining 07.05.2020 yildagi PQ-4708-sonli "Matematika sohasidagi ta'lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora tadbirlari to'g'risida"gi qarorining 1-ilovasidagi 3.1-bo'limining 14- "Matematika bakalavriat ta'lim yo'nalishlari bitiruvchilarining

muayyan aniq sohalarda amaliy masalalarni yechish ko‘nikmalarini rivojlantirish uchun ta’lim dasturlarini fanlar (yo‘nalishlar)aro integrativ prinsip asosida ixtisoslashtirilgan tartibda ishlab chiqish va joriy etish.” bandiga ko’ra bugungi kunga kelib, oliy ta’lim muassasalarida matematika fanini kasbga yo’naltirib o’qitish va hayotiy bog’liqlikda isbotlab o’qitish asosiy vazifalardan biri bo’lib qolmoqda [4]. Shularni inobatga olib ushbu maqolada asosan oliy ta’lim muassasalarining matematika darslari dasturiga kiritilgan aniq integrallar va ularning tadbiqlari mavzusini bir nechta misollardagi tadbiqlarini ko’rib o’tamiz. Aniq integrallar va ularning tadbiqlarini misollardagi tahlilini ko’rib chiqishdan oldin aniq integrallarning aylanma jism hajmini topishdagi bir nechta qo’llash mumkin bo’lgan formulalari bilan tanishib olamiz va misollardagi tadbiqlarini ko’rib chiqamiz.

Bizga $[a,b]$ kesmada aniqlangan va uzluksiz bo’lgan $y=f(x)$ funksiya berilgan bo’lib, $\forall x\in[a,b]$ uchun $f(x)\geq 0$ bo’lsin. Yuqoridan $f(x)$ funksiya grafigi, yon tomonlardan $x=a, x=b$ chiziqlar bilan, pastdan esa Ox o’qdagi $[a,b]$ kesma bilan chegaralangan shaklni Ox o’q atrofida aylanishidan hosil bo’lgan T shaklning hajmi quyidagi (1) formula yordamida hisoblanadi [1]:

$$V=\pi\int_a^b f^2(x)dx \qquad (1)$$

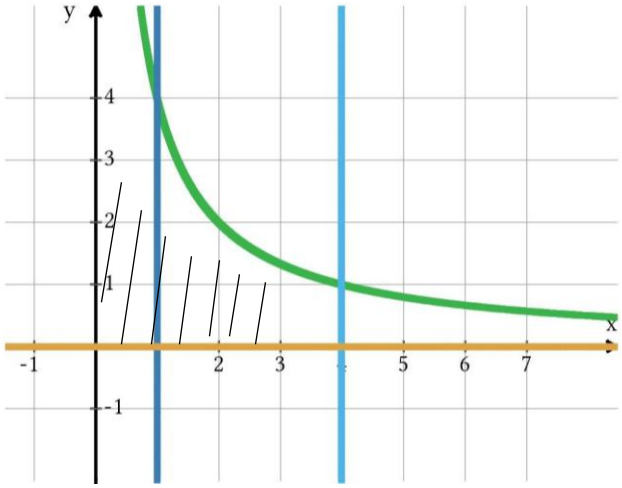
Bizga biror T jism berilgan bo’lib, uning Oy o’qqa parallel bo’lgan kesimlarining yuzasi ma’lum bo’lsin. Bu yuza x o’zgaruvchining funksiyasi bo’ladi, uni $S=S(x)$ orqali belgilaymiz.

Agar $S=S(x)$ funksiya $[a,b]$ kesmada uzluksiz bo’lsa, T shaklning V hajmi quyidagi (2) formula yordamida topiladi [3]:

$$V=\int_a^b S(x)dx \qquad (2)$$

1-misol. Quyidagi $xy = 4, x = 1, x = 4, y = 0$ chiziqlar bilan chegaralangan shaklni Ox o'q atrofida aylantirishdan hosil bo'lgan aylanma jismning hajmini toping [2].

Yechilishi. D soha yuqoridan $xy = 4$ funksiya bilan, yon tomonlardan $x = 1, x = 4$ chiziqlar bilan, pastdan esa Ox $(y = 0)$ o'q bilan chegaralangan (1-rasm).



$y = \frac{4}{x}$

1-rasm

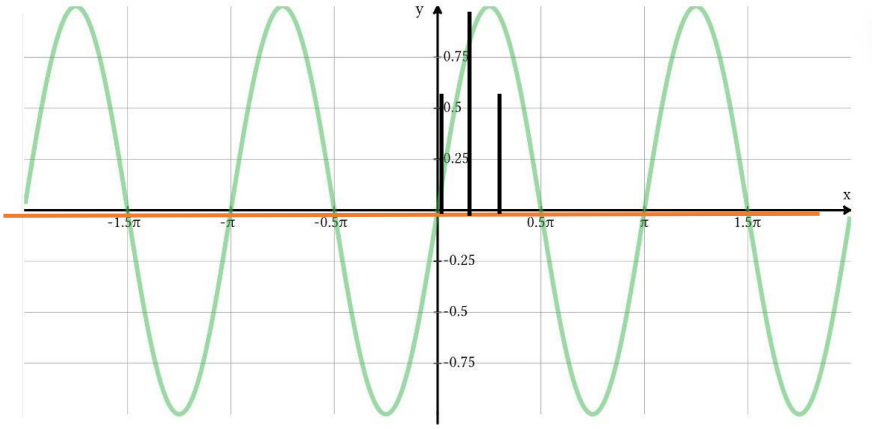
Endi D egri chiziqli sohani Ox $(y = 0)$ o'q atrofida aylantirishdan hosil bo'lgan aylanma jismning hajmini formula yordamida hisoblaymiz:

$$V = \pi \int_a^b y^2 dx = \pi \int_1^4 \frac{16}{x^2} dx = -\pi \left(\frac{16}{x}\Big|_1^4\right) = -\pi(4-16) = 12\pi$$

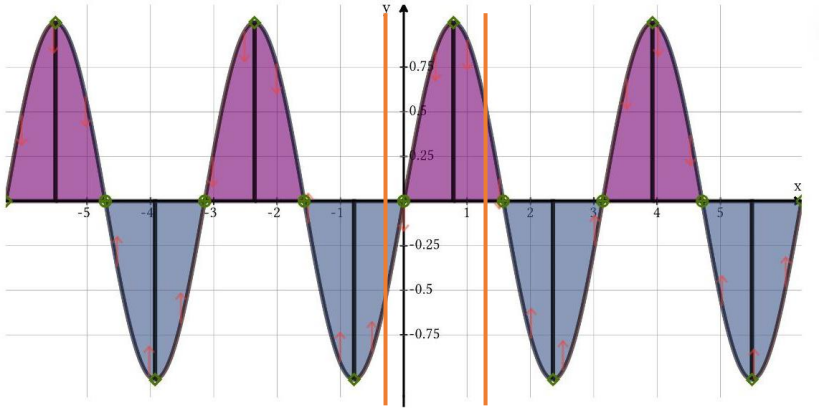
Javob: 12π

2-misol. Quyidagi $y = \sin 2x, 0 \le x \le \frac{\pi}{2}, y = 0$ chiziqlar bilan chegaralangan shaklni Ox o'q atrofida aylantirishdan hosil bo'lgan aylanma jismning hajmini toping [2,3].

Yechilishi. D soha yuqoridan $y = \sin 2x$ funksiya bilan, pastdan esa Ox $(y = 0)$ o'q bilan chegaralangan, $x \in [0; \frac{\pi}{2}]$ (2-rasm).



2-rasm



3-rasm

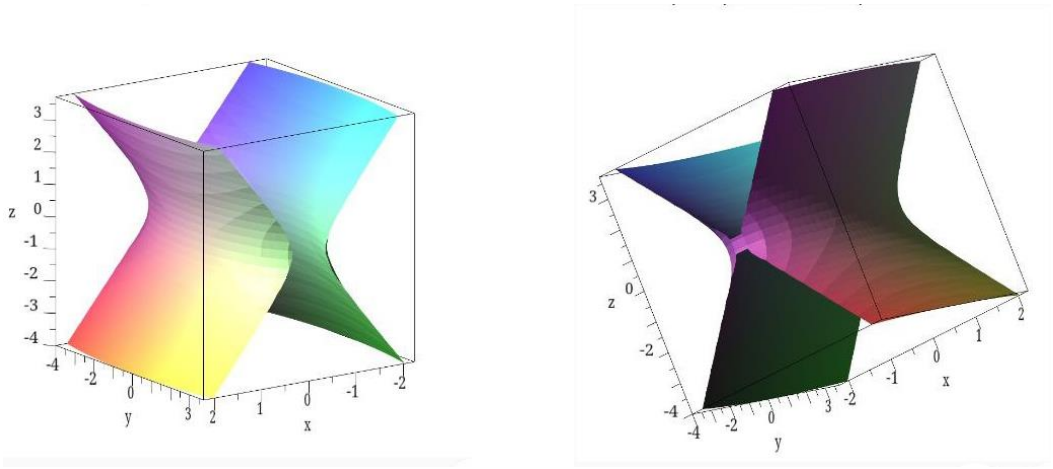
3-rasmda belgilangan D sohani $Ox\ (y=0)$ o'q atrofida aylantirishdan hosil bo'lgan aylanma jismning hajmini formula yordamida hisoblaymiz:

$$V=\pi\int_a^b y^2dx=\quad \pi\int_0^{\frac{\pi}{2}}\sin^2 2xdx=\pi\int_0^{\frac{\pi}{2}}(\frac{1-\cos 4x}{2})dx=\pi((\frac{1}{2}x-\frac{1}{8}\sin 4x)\Big|_0^{\frac{\pi}{2}})=\frac{\pi^2}{4}$$

Javob: $\frac{\pi^2}{4}$

3-misol. Ushbu $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, z=\pm c$ sirt bilan chegaralangan shakl hajmini toping [1,2].

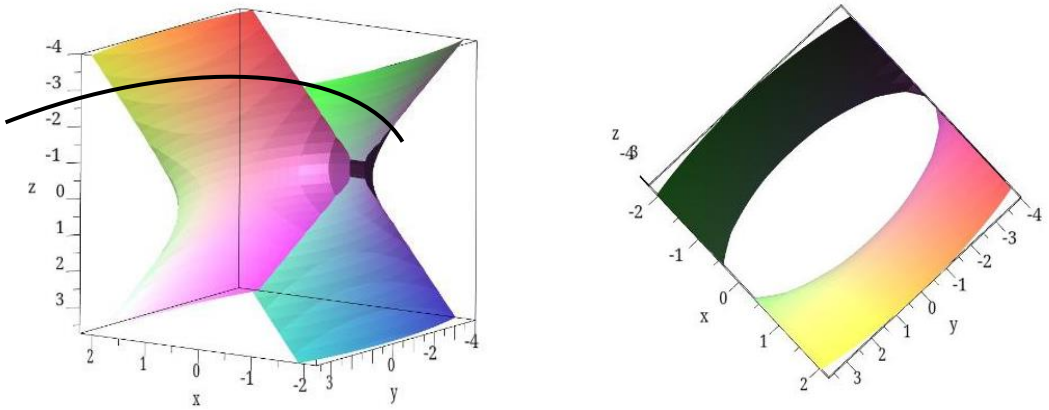
Yechilishi. Dastlab berilgan tenglama bo'yicha bir pallali giperboloidni yasaymiz (4-rasm).



4-rasm

Giperboloidni Oxy tekislikka parallel bo'lgan, $z\epsilon[-c;c]$ kesmada o'zgaruvchi, z=p tekisliklar bilan kesamiz. Kesimda ellips hosil bo'ladi (5-rasm) [5].

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{p^2}{c^2}=1, \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1+\frac{p^2}{c^2}, \quad \frac{x^2}{a_1^{\,2}}+\frac{y^2}{b_1^{\,2}}=1$$



5-rasm

Bunda ellipsning yarim o'qlari:

$$a_1=\frac{a}{c}\sqrt{c^2+p^2} \qquad b_1=\frac{b}{c}\sqrt{c^2+p^2}$$

Bu kesimning yuzlari p ga bog'liq bo'lgan ellips bilan chegaralangan yuzaga teng bo'ladi [5].

$$S(p)=\pi a_1 b_1=\frac{\pi ab}{c^2}(c^2+p^2)$$

Kesim yuzidan foydalanib, berilgan jism hajmini topamiz:

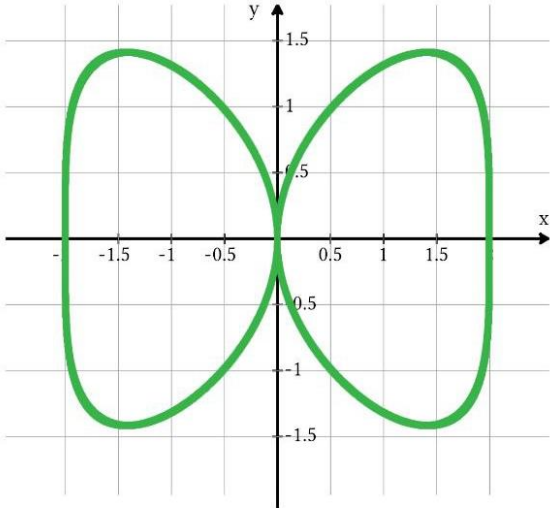
$$V = \int_{-c}^{c} S(p)dp = \int_{-c}^{c} \frac{\pi ab}{c^2}(c^2 + p^2)dp = \frac{2\pi ab}{c^2}(c^2 p + \frac{p^3}{3})\Big|_0^c = \frac{2\pi ab}{c^2} \cdot \frac{4c^3}{3} = \frac{8\pi abc}{3}$$

Javob: $\frac{8\pi abc}{3}$

4-misol. Ushbu $x^4 + y^4 = 4x^2$ egri chiziq bilan chegaralangan figurani Ox o'q atrofida aylantirishdan hosil bo'lgan jism hajmini toping [2].

Yechilishi:

Berilgan egri chiziq tenglamasidan foydalanib, figuraning ko'rinishini hosil qilamiz:



6-rasm

Rasmda (6-rasm) ko'rsatilgan D egri chiziqli sohani Ox o'q atrofida aylantirishdan hosil bo'ladigan jism hajmini hisoblaymiz [5].

Oshkormas ko'rinishda berilgan egri chiziq tenglamasini quyidagi ko'rinishga keltiramiz:

$$y^2 = \sqrt{4x^2 - x^4}$$

Aylanma jism hajmini hisoblash formulasidan foydalanamiz:

$$V = \pi \int_{-2}^{2} y^2 dx = 2\pi \int_{-2}^{0} \sqrt{4x^2 - x^4}\, dx = 2\pi \int_{-2}^{0} x\sqrt{4 - x^2}\, dx = \pi \int_{-2}^{0} \sqrt{4 - x^2}\, d(x^2) = \frac{2\pi}{3}(4 - x^2)^{\frac{3}{2}} \Big|_{-2}^{0} = \frac{16\pi}{3}$$

Javob: $\frac{16\pi}{3}$

XULOSA

Aniq integrallarni ba’zi tadbiqlarini misollar yordamida o’rganish orqali matematika fanini boshqa fanlardagi tutgan o’rnini ham ko’rsatish mumkin. Masalan, ushbu maqolada ko’rib chiqilgan aniq integrallar yordamida aylanma jismlarning hajmini hisoblashga oid misollar nafaqat matematika fanida, balki fizika, kimyo va shu bilan birga texnikaning bir qator muammolarini hal qilishda ham uchrab turadi. Shulardan kelib chiqqan holda ushbu mavzu to’la yoritib berilishi orqali oliy ta’lim muassasalari talabalarining fanga bo’lgan qiziqishi, mantiqiy fikrlashi va muammoni kreativ yondashuv yo’li bilan hal qilish ko’nikmasini shakllantiriladi.

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