UCH O‘LCHAMLI NILPOTENT ALGEBRALARDA LOKAL AVTOMORFIZMLAR
ЛОКАЛЬНЫЕ АВТОМОРФИЗМЫ В ТРЕХМЕРНЫХ НИЛЬПОТЕНТНЫХ АЛГЕБРАХ
LOCAL AUTOMORPHISMS IN THREE-DIMENSIONAL NILPOTENT ALGEBRAS

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Annotatsiya. *Bu ishda uch o’lchovli Nilpotent algebralarda har qanday chiziqli lokal avtomorfizmlar avtomorfizm bo’lishi isboti bilan ko’rsatilgan.*

Аннотация. *В работе доказывается, что любые линейные локальные автоморфизмы в трехмерных нильпотентных алгебрах являются автоморфизмами.*

Abstract. *In this work, it is proved that any linear local automorphisms in three-dimensional Nilpotent algebras are automorphisms.*

Kalit so’zlar: Avtomorfizm, Algebra, lokal avtomorfizm, chiziqli akslatirish.

Ключевые слова: Автоморфизм, алгебра, локальный автоморфизм, линейное отражение.

Key words: Automorphism, Algebra, local automorphism, linear reflection.

Ta’rif. A algebra bo’lib undagi xar bir $x \in A$ element uchun

$$\Phi(x) = \varphi_x(x)$$

shartni qanoatlantiruvchi $\varphi_x : A \to A$ avtomorfizm mavjud bo’lsa, u holda $\Phi : A \to A$ chiziqli akslantirish lokal avtomorfizm deb ataladi.

Quyidagi teoremada A_1 algebra uchun $\Phi : A \to A$ akslantirishni qaraylik.

Teorema. A_1 algebraning har qanday chiziqli lokal avtomorfizmi avtomorfizmdan iborat.

Isbot. Φ A_1 ning ixtiyoriy lokal avtomorfizmi bo'lsin. Barcha $x \in A_1$ uchun ta'rif bo'yicha $x \in A_1$ da $\Phi(x) = \varphi_x(x)$ avtomorfizmi mavjud.

1 teoremaga ko'ra, φ_x avtomorfizmi quyidagi matritsa ko'rinishiga ega:

$$A_x = \begin{pmatrix} \alpha_x & 0 & 0 \\ 0 & \alpha_x^2 & 0 \\ \beta_x & 0 & \alpha_x^4 \end{pmatrix}.$$

Xususan,

$$\Phi(e_1) = \varphi_{e_1} e_1,$$

$$\Phi(e_2) = \varphi_{e_2} e_2,$$

$$\Phi(n_1) = \varphi_{e_3} e_3$$

tengliklarni qanoatlantiruvchi $\varphi_{e_1}, \varphi_{e_2}, \varphi_{e_3}$ matritsalar mavjud. A matritsani quyidagicha quraylik:

$$A = \begin{pmatrix} \alpha_{e_1} & 0 & 0 \\ 0 & \alpha_{e_2}^2 & 0 \\ \beta_{e_1} & 0 & \alpha_{e_3}^4 \end{pmatrix}.$$

Φ chiziqli bo`lgani uchun

$$\Phi(x+y) = \Phi(x) + \Phi(y), \forall\ x, y \in A_1 (¿)$$

tenglik o`rinli.

Bu tenglikka ko`ra

$$\Phi(e_1 + e_2) = \alpha_{e_1+e_2} e_1 + \beta_{e_1+e_2} e_3 + \alpha_{e_1+e_2}^2 e_2,$$

$$\Phi ¿$$

tengliklarga ega bo`lamiz. Bunda bazis elementlarining koeffitsientlarini taqqoslab, biz quyidagilarga erishamiz:

$$\alpha_{e_1+e_2}=\alpha_{e_1},$$
$$\beta_{e_1+e_2}=\beta_{e_1},$$
$$\alpha_{e_1+e_2}=\alpha_{e_2}.$$

Bundan esa $\alpha_{e_1}=\alpha_{e_2}$.

$(¿)$ tenglikdan foydalanib,

$$\Phi(e_2+e_3)=\alpha^2_{e_2+e_3}e_2+\alpha^2_{e_2+e_3}e_3,$$
$$\Phi ¿.$$

Yana bazis elementlarining koeffitsientlarini taqqoslab, biz quyidagilarga erishamiz:

$$\alpha_{e_2+e_3}=\alpha_{e_2},\alpha_{e_2+e_3}=\alpha_{e_3.}$$

Bundan esa $\alpha_{e_2}=\alpha_{e_3}$ bo`ladi.

Shunday qilib, biz Φ lokal avtomorfizm quyidagi shaklga ega ekanligini bilib olamiz:

$$\Phi=\begin{pmatrix}\alpha_{e_1} & 0 & 0\\ 0 & \alpha^2_{e_1} & 0\\ \beta_{e_1} & 0 & \alpha^4_{e_1}\end{pmatrix}$$

Teorema isbotlandi.

FOYDALANILGAN ADABIYOTLAR.

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