IKKI KARRALI INTEGRALNING TATBIQLARI. IKKI KARRALI INTEGRAL YORDAMIDA YUZA VA JISM HAJMINI HISOBLASH. MASSA, O'RTA QIYMAT VA INERSIYA MOMENTI.

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ANNOTATSIYA: Ushbu maqolada matematikaning eng qiziq mavzularidan biri bo'lgan Ikki karrali integralning tatbiqlari. Ikki karrali integral yordamida yuza va jism hajmini hisoblash. Massa, o'rta qiymat va inersiya momentini topish haqida ma'lumotlar berib o'tildi va mavjud muanmolarga ilmiy yondashildi. Ikki karrali integral aniq integralning ikki o'zgaruvchili(argumentli) funksiya uchun umumlashgan holidir. Ikki karrali integral ham aniq integralning asosiy xossalariga ega. Aniq integralning xossalarini takrorlashni tavsiya etamiz.

KALIT SO'ZLAR: Ikki karrali integralning ta'rifi, Ikki karrali integralni hisoblash, Ikki karrali integralning tatbiqlari.

Applications of double integrals. Calculate the volume of a surface and a body using a double integral. Mass, mean and moment of inertia.

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ABSTRACT: In this article, one of the most interesting topics in mathematics is the application of the double integral. Calculate the volume of a surface and a body using a double integral. Information on finding mass, mean and moment of inertia was given, and a scientific approach to the existing problems was given. A double integral is a generalized case of a definite integral for a function with two variables (arguments). The double integral also has the basic properties of a definite integral. We recommend repeating the properties of the exact integral.

KEYWORDS: Definition of double integral, Calculation of double integral, Applications of double integral.

1. Ikki karrali integralning ta'rifi.

$f(x,y)$ funksiya biror D sohada aniqlangan bo'lsin. D sohani n ta D_i qismlarga bo'lamiz. Har bir D_i qismda $P_i(x_i, y_i)$ bittadan nuqta tanlaymiz hamda

$$S_n = \sum_{i=1}^{n} f(x_i, y_i)\Delta S_i \qquad (1)$$

yig'indini to'zamiz. (1) yig'indiga $f(x,y)$ funksiya uchun D sohadagi ***integral yig'indi*** deyiladi. λ qism sohalar diametrlarining eng kattasi bo'lsin. ΔS_i, D_i sohaning yuzi.

Ta'rif. (1) integral yig'indining, qismlarga bo'linish usuliga, P_i nuqtalarning tanlanishiga bog'liq bo'lmagan $\lambda \to 0$ dagi limiti mavjud bo'lsa, bu limitga $f(x,y)$ funksiyaning D sohadagi ***ikki karrali integrali*** deyiladi va

$$\iint_D f(x,y)ds$$

simvol bilan belgilanadi.

Ikki karrali integral aniq integralning ikki o'zgaruvchili(argumentli) funksiya uchun umumlashgan holidir.

Ikki karrali integral ham aniq integralning asosiy xossalariga ega. Aniq integralning xossalarini takrorlashni tavsiya etamiz.

2. Ikki karrali integralni hisoblash.

Ikki karrali integralni hisoblash ikkita aniq integralni ketma-ket hisoblashga keltiriladi. D soha $y = y_1(x)$, $y = y_2(x)$ funksiyalar grafklari hamda $x = a$ *va* $x = b$ to'g'ri chiziqlar bilan chegaralangan bo'lsin, ya'ni

$$\begin{cases} a \le x \le b \\ y_1(x) \le y \le y_2(x) \end{cases}$$

tengsizliklar bilan aniqlangan bo'lsa, ikki karrali integral quyidagicha hisoblanadi:

$$\iint_D f(x,y)ds = \int_a^b \left[\int_{y_1(x)}^{y_2(x)} f(x,y)dy \right] dx = \int_a^b dx \int_{y_1(x)}^{y_2(x)} f(x,y)dy \qquad (1)$$

Oxirgi aniq integral ***ichki integral*** deb ataladi va uni hisoblashda x ni o'zgarmas deb, integrallash y bo'yicha olib boriladi. Ichki integralni hisoblash natijasi ***tashqi integral*** uchun integral osti funksiyasi bo'ladi.

D soha

$$\begin{cases} c \le y \le d \\ x_1(y) \le x \le x_2(y) \end{cases}$$

tengsizliklar bilan aniqlangan bo'lsa , ikki karrali integral

$$\iint_D f(x,y)dxdy = \int_s^d \left[\int_{x_1(y)}^{x_2(y)} f(x,y)dx \right] dy = \int_s^d dy \int_{x_1(y)}^{x_2(y)} f(x,y)dx$$

formula yordamida ikkita aniq integralni hisoblashga keltiriladi.

1-misol. $\iint_D x\ln y\,dxdy$ integralni D soha: $0\le x\le 4$, $1\le y\le e$ to’g’ri to’rtburchak bo’lganda hisoblang.

Yechish. (1) formulaga asosan,

$$\iint_D x\ln y\,dxdy=\int_0^4 xdx\int_0^e \ln y\,dy=\int_0^4 xdx\left[y\ln y-y\right]_1^e=\frac{x^2}{2}\Big|_0^4=8.$$

2-misol. $\iint_D (x-y)dxdy$ integralni $D: y=2-x^2,\ \ y=2x-1$, chiziqlar bilan chegaralangan soha bo’lganda hisoblang.

Yechish. Birinchi chiziq uchi (0,2) nuqtada OY o’qiga simmetrik bo’lgan parabola. Ikkinchisi chiziq to’g’ri chiziq. Bu chiziqlarning kesishish nuqtalarini topamiz:

$$\begin{cases} y=2-x^2 \\ y=2x-1 \end{cases}$$

tenlamalar sistemasini yechib, $A(-3;-7)$, $B(1,1)$ nuqtalarni topamiz. (1) formulaga asosan,

$$\iint_D (x,y)dxdy=\int_{-3}^{1} dx\int_{2x-1}^{2-x^2}(x-y)dy=$$

$$=\int_{-3}^{1}\left[xy-\frac{y^2}{2}\right]_{2x-1}^{2-x^2}dx=\int_{-3}^{1}\left[x\cdot(2-x^2)-\frac{(2-x^2)^2}{2}-\left[x\cdot(2x-1)-\frac{(2x-1)^2}{2}\right]\right]dx=$$

$$=\int_{-3}^{1}\left(2x-x^3-\frac{4-4x^2+x^4}{2}-2x^2+x+\frac{4x^2-4x+1}{2}\right)dx=$$

$$\int_{-3}^{1}$$

$$=\int_{-3}^{1}\left(-\frac{1}{2}x^4-x^3+2x^2+x-\frac{3}{2}\right)dx=$$

$$=\left[-\frac{1}{10}x^5-\frac{1}{4}x^4+\frac{2}{3}x^3+\frac{1}{2}x^2-\frac{3}{2}x\right]_{-3}^{1}=4\frac{4}{15}$$

bo’ladi.

2. Ikki karrali integralning tatbiqlari.

1. $\iint_D f(x,y)dxdy$ integralda $f(x,y)=1$ bo’lsa, $\iint_D dxdy$ integral D figuraning yuzini ifodalaydi, ya’ni

$$S=\iint_D dxdy$$

1-misol. $x=4y-y^2$, $x+y=6$ chiziqlar bilan chegaralangan sohaning yuzini toping.

Yechish. Berilgan chiziqlarning kesishish nuqtalarini topamiz. $x=4y-y^2,\ x=6-y\quad dan\quad 4y-y^2=6-y,\quad y^2-5y+6=0,$ $y_1=2,\ y_2=3;\ x_1=4,\ x=3;\ A(4;2)\ va\ B(3;3)$ kesishish nuqtalari bo'ladi.

Shunday qilib, yuza

$$S=\iint_D dxdy=\int_2^3 dy\int_{6-y}^{4y-y^2}dx=\int_2^3 x\Big|_{6-y}^{4y-y^2}dy=\int_2^3\left(4y-y^2-6+y\right)dy=\int_2^3\left(5y-y^2-6\right)dy=$$

$$=\left(\frac{5}{2}y^2-\frac{y^3}{3}-6y\right)\Bigg|_2^3=\frac{1}{6}\ \text{(kv. birlik)}$$

2. Yuqoridan $z=f(x,y)$ sirt, quyidan $z=0$ tekislik, yon tomondan to'g'ri silindrik sirt bilan hamda XOY tekislikda D sohani hosil qiladigan ***silindrik jismning xajmi***

$$V=\iint_D f(x,y)dxdy\quad \text{integral bilan xisoblanadi.}$$

2-misol. $y=1+x^2,\ z=3x, y=5, z=0$ sirtlar bilan chegaralangan I oktantadagi jismning hajmini hisoblang.

Yechish. Hajmi hisoblanishi kerak bo'lgan jism yuqoridan $z=3x$ tekislik, yondan $y=1+x^2$ parabolik silindr, $y=5$ tekislik bilan chegaralangan. Shunday kilib

$$V=\iint_D 3xdxdy=3\int_0^2 xdx\int_{1+x^2}^{5}dy=3\int_0^2 x\left[5-(1+x^2)\right]dx=3\int_0^2\left(4x-x^3\right)dx=3\left(4\frac{x^2}{2}-\frac{x^4}{4}\right)\Bigg|_0^2=$$ 3.

$$=3\left(2\cdot2^2-\frac{2^4}{4}\right)=24-12=12\ куб.бир.$$

Plastinka har bir nuqtasidagi zichlik funksiyasi $\gamma(x,y)$ bo'lsa, uning massasi

$$m=\iint_D \gamma(x,y)dxdy$$

integral bilan hisoblanadi.

Plastinkaning $OX\ \ va\ \ OY$ o'qlarga nisbatan ***statik momentlari*.**

$$M_x=\iint_D y\gamma(x,y)dxdy\ ,\ M_y=\iint_D x\gamma(x,y)dxdy$$

formulalar bilan hisoblanadi.

Plastinka birjinsli, ya'ni $\gamma=\cos nt$ bo'lganda uning ***og'irlik markazining koordinatalari***

$$\bar{x}_c=\frac{M_x}{S}=\frac{\iint xdxdy}{S}\qquad \bar{y}_c=\frac{M_x}{S}=\frac{\iint_D ydxdy}{S}$$

formulalar yordamida topiladi, bu yerda S, D sohaning yuzi.

Plastinkaning OX va OU o'qlariga nisbatan ***inertsiya momentlari***

$$J_x = \iint_D y^2\gamma(x,y)dxdy\ ,\ J_y = \iint_D x^2\gamma(x,y)dxdy$$

formulalar bilan, koordinatlar boshiga nisbatan inertsiya momenti

$$J_0 = \iint_D \left(x^2 + y^2\right)\gamma(x,y)dxdy = J_x + J_y$$

formula bilan aniqlanadi. Yuqoridagi formulalarda $\gamma(x,y) = 1$ deb tekis figuralarning geometrik inertsiya momentlarini topish formulalarini olamiz.

3-misol. $y^2 = 4x + 4,\ \ y^2 = -2x + 4$ chiziqlar bilan chegaralangan figuraning og'irlik markazining koordinatlarini toping.

Yechish. Chiziqlar OX o'qiga nisbatan simmetrik bo'lganligi uchun $\bar{y}_c = 0$ $\bar{x}_c$ ni topamiz:

$$S = \iint_D dxdy = 2\int_0^2 dy \int_{\frac{y^2-4}{4}}^{\frac{4-y^2}{2}} dy = 2\int_0^2 \left(\frac{4-y^2}{2} - \frac{y^2-4}{4}\right)dy = 2\int_0^2 \left(3 - \frac{3y^2}{4}\right)dy =$$

$$= 6\left[y - \frac{y^3}{12}\right]_0^2 = 6\left(2 - \frac{8}{12}\right) = 8$$

$$\bar{x}_c = \frac{1}{8}\iint_L xdxdy = \frac{1}{8}2\int_0^2 dy \int_{\frac{y^2-4}{4}}^{\frac{4-y^2}{2}} xdxdy = \frac{1}{8}\int_0^2 \left[\frac{4-y^2}{4} - \frac{\left(y^2-4\right)}{16}\right]dy = \frac{1}{8}\int_0^2 \left(3 - \frac{3}{2}y^2 + \frac{3}{16}y^4\right)dy =$$

$$= \frac{1}{8}\left(3y - \frac{y^3}{2} + \frac{3y^5}{80}\right)_0^2 = \frac{2}{5}.\quad \text{Demak } C(\frac{2}{5};0).$$

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