BIRINCHI VA IKKINCHI TUR EGRI CHIZIQ INTEGRALLAR (GEOMETRIK VA FIZIK MA'NOLARI.) GRIN FORMULASI.

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ANNOTATSIYA: Ushbu maqolada matematikaning eng qiziq mavzularidan biri bo'lgan I-va II-tur egri chiziq integrallar (geometrik va fizik ma'nolari.) Grin formulasi haqida ma'lumotlar berib o'tildi va mavjud muanmolarga ilmiy yondashildi hamda muanmolarni hal etish uchun tegishli tavsiyalar berib o'tildi.

KALIT SO'ZLAR : I tur egri chiziq integrali uni hisoblash va xossalari, II tur egri chiziqli integrallar , xossalari va hisoblash, Grin formulasi.

The first and second types of curvilinear integrals (geometric and physical meanings.) Green's formula.

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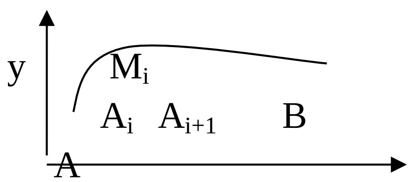
ABSTRACT: This article provides information about the Green formula, one of the most interesting topics in mathematics, type I and II curvilinear integrals (geometric and physical meanings), and provides a scientific approach to existing problems and to solve problems. appropriate recommendations were made.

KEYWORDS: Type I curve integral, its calculation and properties, Type II curvilinear integrals, properties and calculation, Green's formula.

1. I tur egri chiziq integrali uni hisoblash va xossalari.

Tekislikda biror silliq AB egri chiziq berilgan bo'lib, unda f(x,y) funksiya aniqlangan bo'lsin.



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Endi AB egri chiziqni $A=A_0, A_1, A_2, ..., A_n=B$ nuqtalar bilan $A_i A_{i+1}$ $(i=0,...,n-1)$ yoylarga ajratamiz, har bir yoychada ixtiyoriy $M_i(\xi_i,\eta_i)$ nuqta olib bu nuqtadagi f(x,y) funksiyani qiymatini $f(\xi_i,\eta_i)$ deb belgilab quyidagi yig'indini tuzamiz.

$$\sigma=\sum_{i=0}^{n-1} f(\xi_i,\eta_i)\Delta s_i \qquad (1)$$

$\max\{\Delta s_i\}=\lambda$ deb belgilaylik.

Ta'rif: Agar AB egri chiziqda aniqlangan f(x,y) funksiya uchun tuzilgan (1) yig'indi $\lambda\to 0$ da AB egri chiziqni A_i, A_{i+1} yoylarga bo'lish usuliga va har bir A_i, A_{i+1} yoychada $M_i(\xi_i,\eta_i)$ nuqtani tanlab olish usuliga bog'liq bo'lmagan limitga ega bo'lsa , bu limitga f(x,y) funksiyadan AB egri chiziq bo'yicha olingan birinchi tip egri chiziqli integral deyiladi va

$\int\limits_{(AB)} f(x,y)ds$ deb belgilanadi. Demak. $\lim\limits_{\lambda\to 0}\sigma=\lim\limits_{\lambda\to 0}\sum\limits_{i=0}^{n-1} f(\xi_i,\eta_i)\Delta s_i = \int\limits_{(AB)} f(x,y)ds$

Birinchi tur egri chiziqli integralning asosiy xossalari:

1. $\int\limits_{(AB)} f(x,y)ds=\int\limits_{(BA)} f(x,y)ds$
2. $\int\limits_{(AB)} Cf(x,y)ds = C\int\limits_{AB} f(x,y)ds$
3. $\int\limits_{(k)}[f_1(x,y)\pm f_2(x,y)]ds=\int\limits_{(k)} f_1(x,y)ds\pm\int\limits_{(k)} f_2(x,y)ds$
4. Agar $k=k_1+k_2$ bo'lsa, $\int\limits_{(k)} f(x,y)ds=\int\limits_{(k_1)} f(x,y)ds+\int\limits_{(k_2)} f(x,y)ds$ bo'ladi.

Agar birinchi tur egri chiziqli integralda f(x,y)=1 desak , u xolda $\int\limits_{(k)} f(x,y)ds=\int\limits_{(k)}\mathrm{ds}=\mathrm{S}$ -egri chiziqning uzunligini beradi. Agar f(x,y) funksiyani musbat va o'zgaruvchan chiziqli zichlik $\gamma=f(x,y)$ deb qarasak, $\int\limits_{(k)} f(x,y)ds$ - integral k-egri chiziqning massasini ifodalaydi.

Teorema. Agar f(x,y) funksiya, parametrik tenglamasi

$$\left.\begin{aligned} x&=\varphi(t)\\ y&=\varphi_2(t)\end{aligned}\right\} \text{булган}(\alpha\le t\le\beta)$$

(k) egri chiziqda aniqlangan va uzluksiz bo'lsa, u xolda

$\int\limits_{(k)} f(x,y)ds$ integral mavjud bo'lib

$$\int\limits_{(k)} f(x,y)ds=\int\limits_{\alpha}^{\beta} f[\varphi(t),\psi,(t)]\sqrt{\varphi^{1^2}(t),+\psi^{1^2}(t)}dt$$

formula bilan hisoblanadi.

Agar fazodagi (k) egri chiziqning tenglamasi

$x=\varphi_1(t), y=\varphi_2(t), z=\varphi_3(t) \qquad (t_1\le t\le t_2)$ bo'lsa,

$$\int\limits_{(k)} f(x,y,z)ds = \int\limits_{t_1}^{t_2} f[\varphi_1(t),\varphi_2(t),\varphi_3(t)]\sqrt{\varphi_1'^2(t)+\varphi_2'^2(t)+\varphi_3'^2(t)}dt$$ bo'ladi.

Misol. Zichligi $p = f(x,y,z) = \sqrt{2y}$ qonun bilan o'zgaradigan va fazodagi parametrik tenglamasi $x = t, y = \frac{1}{2}t^2, t = \frac{1}{3}t^3$ $(0 \le t \le 1)$ lar bilan berilgan egri chiziqning massasini toping.

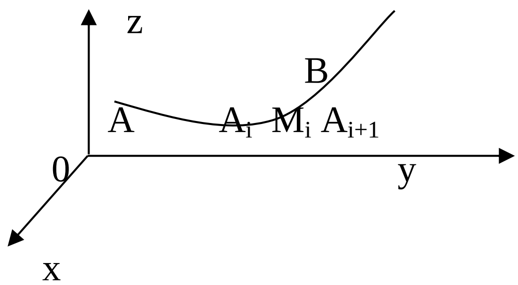
$$M = \int\limits_{(k)} f(x,y,z)ds = \int\limits_k pds = \int\limits_k \sqrt{2y}ds = \int\limits_0^1 \sqrt{2\cdot\frac{1}{2}\cdot t^2}\sqrt{x^{1^2}+y^{1^2}+z^{1^2}}dt = \int\limits_0^1 t\sqrt{1+t^2+t^4}dt =$$

$$= \frac{1}{2}\int\limits_0^1 \sqrt{\left(t^2+\frac{1}{2}\right)^2+\frac{3}{4}}d\left(t^2+\frac{1}{2}\right) = \frac{1}{2}\left[\frac{t^2+\frac{1}{2}}{2}\cdot\sqrt{t^4+t^2+1}+\frac{3}{8}en(t^2+\frac{1}{2}+\sqrt{t^4+t^2+1})\right]_0^1 =$$

$$= \frac{1}{8}\left(3\sqrt{3}-1+\frac{3}{2}en\frac{3+2\sqrt{3}}{3}\right)$$

2.Ikkinchi tur egri chiziqli integral.

Fazoda aniq yo'nalishli silliq (gladkoy) AB egri chiziq berilgan bo'lib, unda P(x,y,z), Q (x,y,z) va R (x,y,z) funksiyalar aniqlangan bo'lsin odatdagicha bu egri chiziqni $A=A_0$, A_1, ..., A_{n-1}, $A_n=B$ nuqtalar bilan $A_i A_{i+1}$ yoylarga ajratib har bir $A_i A_{i+1}$ yoychada ixtiyoriy M(ξ_i,η_i,ζ_i) nuqta olib quyidagicha yig'indi tuzamiz.



$$\sum_{i=0}^{n-1}[(\xi_i,\eta_i,\zeta_i)\Delta x_i + Q(\xi_i,\eta_i,\zeta_i)\Delta y_i + R(\xi_i,\eta_i,\zeta_i)\Delta z] \qquad (2)$$

$\Delta x_i, \Delta y_i$ va Δz_i lar $A_i A_{i+1}$ yoyning mos ravishda ox,oy, va oz o'qlariga bo'lgan proeksiyasi $\max\{\Delta x_i\}=\lambda_1$,$\max\{\Delta y_i\}=\lambda_2$, $\max\{\Delta z_i\}=\lambda_3$, deylik

Ta'rif. Agar AB da aniqlangan P (x,y,z), Q (x,y,z) va R (x,y,z) funksiyalar uchun tuzilgan (2) integral yig'indi $\lambda_1 \to 0, \lambda_2 \to 0, \lambda_3 \to 0$ da AB egri chiziqni $A_i A_{i+1}$ yoylarga va har bir $A_i A_{i+1}$ yoyda ixtiyoriy M(ξ_i,η_i,ζ_i),nuqtani tanlab olish usuliga botsliq bo'lmagan limitga ega bo'lsa bu limitga ' (x,y,z), Q (x,y,z) va R (x,y,z) funksiyalardan AB egri chiziq bo'ylab A dan V ga qarab olingan ikkinchi tip egri chiziqli integral deyiladi va

$$\int\limits_{AB} P(x,y,z)dx + \int\limits_{AB} Q(x,y,z)dy + \int\limits_{AB} R(x,y,z)dz$$ yoki

$$\int\limits_{AB} P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz$$ ko'rinishda yoziladi.

Demak $\int\limits_{AB} Pdx + Qdy + Rdz = \lim\limits_{\substack{\lambda_1 \to 0 \\ \lambda_2 \to 0 \\ \lambda_{31} \to 0}} \sum\limits_{i=1}^{n-1} [P\,\Delta x_i + Q\Delta y_i + R\Delta z_i]$

Agar (2) integral yig'indini P,Q,R funksiyalarning ixtiyoriy bittasi yoki ixtiyoriy ikkitasi uchun tuzsak u xolda ikkinchi tip egri chiziqli integralimiz quyidagi ko'rinishlarda bo'ladi.

$$\int\limits_{AB} Pdx + Qdy, \int\limits_{AB} Qdy + Rdz, \int\limits_{AB} Pdy + Rdz, \int\limits_{AB} Pdy, \int\limits_{AB} Qdy, \int\limits_{AB} Rdz$$

Agar P(x,y,z), Q (x,y,z), R (x,y,z) funksiyalarni $\vec{F}$ kuchning ox, oy, oz o'qlaridagi proek**ts**iyasi sifatida qarasak va Δx, Δy, Δz larni AB egri chiziqning F kuch tahsir qilayotgan nuqtasining ko'chishi Δs ning ox, oy, oz o'qlaridagi proek**s**iyasi sifatida qarasak, u xolda ikkinchi tur egri chiziqli integral $\vec{F}$ kuchning butun AB egri chiziq bo'ylab bajargan ishni beradi, ya'ni A= $\int\limits_{AB} Pdy + Qdy + Rdz$ bo'ladi.

Ikkinchi tip egri chiziqli integralda integrallash yo'nalishini o'zgartirsak, integral qiymati o'z ishorasini o'zgartiradi

$\int\limits_{AB} P(x,y,z)dx = -\int\limits_{BA} P(x,y,z)dx$ chunki Δx_1 ning ishorasi o'zgaradi.

Ikkinchi tip egri chiziqli integralning qolgan hossalari esa birinchi tip egri chiziqli integralning xossalari kabi bo'ladi.

Teorema. AB egri chiziqning tenglamasi parametrik xolda berilgan bo'lib: $\left.\begin{matrix} x = \varphi_1(t) \\ y = \varphi_2(t) \\ z = \varphi_3(t) \\ \alpha \le t \le \beta \end{matrix}\right\}$

(x,y,z) nuqta A dan B ga qarab harakat qilsin.

Agar $\varphi_1(t),\ \varphi_2(t),\ \varphi_3(t)$, P(x,y,z),Q(x,y,z), R(x,y,z) funksiyalar AB da uzluksiz va uzluksiz $\varphi_1^1(t),\ \varphi_2^1(t),\ \varphi_3^1(t)$, hosilalarga ega bo'lsa, u xolda $\int\limits_{AB} Pdx + Qdy + Rdz$ ikkinchi tur egri chiziqli integral mavjud va $\int\limits_{AB} P(x,y,z)dx = \int\limits_{\alpha}^{\beta} [\varphi_1(t), \varphi_2(t)\varphi_3(t)]\varphi_1^1(t)dt$ teng bo'ladi.

Misol. Agar AB egri chiziqning parametrik tenglamasi

$\left.\begin{matrix} x = \sqrt{\cos t} \\ y = \sqrt{\sin t} \\ 0 \le t \le \frac{\pi}{2} \end{matrix}\right\}$ bo'lsa, $\int\limits_{AB} x^2 ydy - y^2 xdx$ integralni hisoblang

Yechimi. $dx = -\frac{\sin t}{2\sqrt{\cos t}}dt$, $dy = -\frac{\cos t}{2\sqrt{\sin t}}dt$ -bularni va x,y larni berilgan interalga qo'ysak.

$$\int_{AB} x^2 ydy - y^2 xdx = \int_0^{\pi/2} \left(\cos t \cdot \sqrt{\sin t} \cdot \frac{\cos t}{2\sqrt{\sin t}} + \sin t \cdot \sqrt{\cos t} \cdot \frac{\sin t}{2\sqrt{\cos t}} \right) dt =$$

$$= \frac{1}{2} \int_0^{\pi/2} (\cos^2 t + \sin^2 t) dt = \frac{1}{2} \int_0^{\pi/2} dt = \frac{\pi}{4} .$$

Misol. Tenglamasi $\left. \begin{aligned} x &= a\cos t \\ y &= b\sin t \end{aligned} \right\}$ parametrik ko'rinishda berilgan ellipsning yuzi hisoblansin.

$$S = \frac{1}{2} \int_Z xdy - ydx = \frac{1}{2} \int_0^{2\pi} [a\cos t \cdot b\cos t + b\sin t \cdot a\sin t] \quad dt =$$

$$= \frac{ab}{2} \int_0^{2/\pi} (\cos^2 t + \sin^2 t) dt = ab\pi$$

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