UCH O'LCHAMLI NILPOTENT ALGEBRANING DIFFERENSIALLASHI LOKAL DIFFERENSIALLASHI

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Izoh: *Differensiallash matematikaning fundamental tushunchalaridan biri hisoblanadi. Differensiallashlar algebra fanida ham muhim o'rin tutadi. Differensiallashlarning turli umumlashmalari mavjud. Bular sarasiga antidifferensiallashlar, δ-differensiallashlar, ternar differensiallashlar va (α,β,γ)-differensiallashlar kiradi. Ushbu ishda uch o'lchamli nilpotent algebralarning differensiallashi lokal differensiallashi isboti bilan ko'rsatilgan.*

Kalit so'zlar: Differensiallash, Algebra, lokal differensiallash, chiziqli akslatirish.

ДИФФЕРЕНЦИАЦИЯ ТРЕХМЕРНОЙ НИЛЬПОТЕНТНОЙ АЛГЕБРЫ ЛОКАЛЬНАЯ ДИФФЕРЕНЦИАЦИЯ

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Аннотация. Дифференциация — одно из фундаментальных понятий математики. Дифференциации также играют важную роль в алгебре. Существуют различные обобщения дифференциаций. К ним относятся антидифференциации, δ-дифференциации, тернарные дифференциации и (α,β,γ)-дифференциации. В данной работе дифференциация трехмерных нильпотентных алгебр демонстрируется путем доказательства локальной дифференциации.

Ключевые слова: Дифференциация, алгебра, локальное дифференцирование, линейное отображение.

DIFFERENTIATION OF THREE-DIMENSIONAL NILPOTENT ALGEBRA LOCAL DIFFERENTIATION

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Abstract. Differentiation is one of the fundamental concepts of mathematics. Differentiations also play an important role in algebra. There are various generalizations of differentiations. These include antidifferentiations, δ-differentiations, ternary differentiations, and (α,β,γ)-differentiations. In this work, the differentiation of three-dimensional nilpotent algebras is shown by proving local differentiation.

Key words: Differentiation, Algebra, local differentiation, linear mapping.

I. KIRISH

1-§. Masalaning qo‘yilishi

Hozirgi kunda jahonda differensiyalashlar nazariyasi bilan bir qatorda, operator algebralarida lokal va 2-lokal differensiyalashlar nazariyasi ham muhim hisoblanadi. So’nggi 20 yil davomida fon Neyman algebralari C^* - algebralari va JB^* - uchliklarda lokal va 2- lokal differensiallashlar nazariyasini o’rganish bo’yicha samarali natijalarga erishildi. Lokal differensiallashlarni o’rganish 1990-yilda R.B.Kadison va D.R.Larson hamda A.S.Sururlar tomonidan boshlangan.

Ixtiyoriy uch o’lchamli nilpotent algebra bazislari $\{e_1, e_2, e_3\}$ bo’lgan quyidagi o’zaro izomorf bo’lmagan algebralarning biriga izomorf bo’ladi:

N_1: $e_1^2=e_2$,$e_2^2=e_3$ N_2: $e_1^2=e_2$, $e_2e_1=e_3, e_2^2=e_3$

$N_3: e_1^2=e_2, e_2e_1=e_3$ $N_4(\alpha): e_1^2=e_2, e_1e_2=e_3, e_2e_1=\alpha e_3$

$N_5: e_1^2 = e_2$ $N_6: e_1^2 = e_3, e_2^2 = e_3$

$N_7: e_1 e_2 = e_3, e_2 e_1 = -e_3$ $N_8(\alpha): e_1^2 = \alpha e_3, e_2 e_1 = e_3, e_2^2 = e_3$

II.ADABIYOTLAR TAHLILI

Differensiallashning ko'plab umumlashmalari mavjud. Eng asosiy umumlashmasi bu lokal va 2-lokal umumlashmalari hisoblanadi. So'ngi yillarda ko'plab olimlar lokal va 2- lokal differensiallashlarga oid ko'plab maqolalar chop etishdi[1- 3] operator algebralarida lokal va 2-lokal differensiyalashlar nazariyasi ham muhim hisoblanadi

III.TADQIQOT METODOLOGIYASI

R haqiqiy sonlar maydoni ustida aniqlangan $\{e_1, e_2, e_3\}$ bazisli nilpotent algebra N_3 berilgan. Bu algebrada ko'paytirish jadvali quyidagicha aniqlanadi:

$$e_1^2 = e_2, e_2 e_1 = e_3$$

N_3 algebrada ihtiyoriy x vektorni

$$x = x_1 e_1 + x_2 e_2 + x_3 e_3$$

ko'rinishda yozishmumkin. $d: N_3 \rightarrow N_3$ akslantirishni qaraylik.

$d(x) = Ax$ chiziqli akslantirishni qaraylik, bu yerda $A - 3 \times 3$ o'lchamli kvadrat matritsa. Ravshanki, agar N_3 algebaradan olingan ixtiyoriy x, y elementlar uchun

$$A(xy) = (Ax)y + x(Ay)$$

Tenglik bajarilsa $d(x) = Ax$ akslantirish qaralayotgan N_3 algebrada differensiallashdan iborat bo'ladi.

1-Teorema. $d(x) = Ax$ akslantirish N_3 algebrada differensiallashdan iborat bo'lishi uchun A matritsa

$$A = \begin{pmatrix} a_{1,1} & 0 & 0 \\ a_{2,1} & 2a_{1,1} & 0 \\ a_{3,1} & a_{2,1} & 3a_{1,1} \end{pmatrix} (4)$$

ko'rinishda bo'lishi zarur va yetarli.

Isbot. Zarurligi. $d(x) = Ax$ akslantirish N_3 algebrada differensiallashdan iborat bo'sin. U holda

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}$$

Matritsa ixtiyoriy x, y vektorlar uchun (1) tenglikni qanoatlantirishi kerak.

Xususan $e_1 e_1 = e_2$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_1)e_1 + e_1(Ae_1) = A(e_2)$$

qaraylik:

$$Ae_1 = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{3,1} \end{pmatrix} = a_{1,1} e_1 + a_{2,1} e_2 + a_{3,1} e_3$$

$$Ae_2 = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{1,2} \\ a_{2,2} \\ a_{3,2} \end{pmatrix} = a_{1,2} e_1 + a_{2,2} e_2 + a_{3,2} e_3$$

$$(Ae_1)e_1 = (a_{1,1} e_1 + a_{2,1} e_2 + a_{3,1} e_3)e_1 = a_{1,1} e_2 + a_{2,1} e_3$$

$$e_1(Ae_1) = e_1(a_{1,1} e_1 + a_{2,1} e_2 + a_{3,1} e_3) = a_{1,1} e_2$$

$$a_{1,1} e_2 + a_{2,1} e_3 + a_{1,1} e_2 = a_{1,2} e_1 + a_{2,2} e_2 + a_{3,2} e_3$$

bundan

$$a_{2,2} = 2a_{1,1}, a_{1,2} = 0, a_{3,2} = a_{2,1}$$

$$Ae_2 = 2a_{1,1} e_2 + a_{2,1} e_3$$

tengliklarni aniqlaymiz.

Endi $e_1 e_2 = 0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_1)e_2 + e_1(Ae_2) = A(0)$$

qaraylik:

$$(Ae_1)e_2 = (a_{1,1} e_1 + a_{2,1} e_2 + a_{3,1} e_3)e_2 = 0$$

$$e_1(Ae_2) = e_1(2a_{1,1} e_2 + a_{2,1} e_3) = 0$$

$$0 = 0$$

bo‘ldi.

Endi $e_1 e_3 = 0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_1)e_3 + e_1(Ae_3) = A(0)$$

qaraylik:

$$(Ae_1)e_3 = (a_{1,1} e_1 + a_{2,1} e_2 + a_{3,1} e_3)e_3 = 0$$

$$e_1(Ae_3) = e_1(a_{1,3} e_1 + a_{2,3} e_2 + a_{3,3} e_3) = a_{1,3} e_2$$

$$0 + a_{1,3} e_2 = 0$$

bundan

$$a_{1,3}=0, A e_3=a_{2,3} e_2+a_{3,3} e_3$$

tengliklarni aniqlaymiz.

Endi $e_2 e_1=e_3$ tenglikdan foydalanib quyidagi ayniyatni

$$(A e_2) e_1+e_2 (A e_1)=A(e_3)$$

qaraylik:

$$(A e_2) e_1=(2 a_{1,1} e_2+a_{2,1} e_3) e_1=2 a_{1,1} e_3$$

$$e_2 (A e_1)=e_2 (a_{1,1} e_1+a_{2,1} e_2+a_{3,1} e_3)=a_{1,1} e_3$$

$$2 a_{1,1} e_3+a_{1,1} e_3=a_{2,3} e_2+a_{3,3} e_3$$

bundan

$$a_{2,3}=0, a_{3,3}=3 a_{1,1}, A e_3=3 a_{1,1} e_3$$

tengliklarni aniqlaymiz.

Endi $e_2 e_2=0$ tenglikdan foydalanib quyidagi ayniyatni

$$(A e_2) e_2+e_2 (A e_2)=A(0)$$

qaraylik:

$$(A e_2) e_2=(2 a_{1,1} e_2+a_{2,1} e_3) e_2=0$$

$$e_2 (A e_2)=e_2 (2 a_{1,1} e_2+a_{2,1} e_3)=0$$

$$0=0$$

bo‘ldi.

Endi $e_2 e_3=0$ tenglikdan foydalanib quyidagi ayniyatni

$$(A e_2) e_3+e_2 (A e_3)=A(0)$$

qaraylik:

$$(A e_2) e_3=(2 a_{1,1} e_2+a_{2,1} e_3) e_3=0$$

$$e_2 (A e_3)=e_2 3 a_{1,1} e_3=0$$

$$0=0$$

bo‘ldi.

Endi $e_3 e_1=0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_3)e_1+e_3(Ae_1)=A(0)$$

qaraylik:

$$(Ae_3)e_1=3a_{1,1}e_3e_1=0$$

$$e_3(Ae_1)=e_3(a_{1,1}e_1+a_{2,1}e_2+a_{3,1}e_3)=0$$

$$0=0$$

bo‘ldi.

Endi $e_3e_2=0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_3)e_2+e_3(Ae_2)=A(0)$$

qaraylik:

$$(Ae_3)e_2=3a_{1,1}e_3e_2=0$$

$$e_3(Ae_2)=e_3(2a_{1,1}e_2+a_{2,1}e_3)=0$$

$$0=0$$

bo‘ldi.

Endi $e_3e_3=0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_3)e_3+e_3(Ae_3)=A(0)$$

qaraylik:

$$(Ae_3)e_3=3a_{1,1}e_3e_3=0,e_3(Ae_3)=e_33a_{1,1}e_3=0$$

$$0=0$$

bo‘ldi.

Yetarliligi. Endi A matritsa (4) ko‘rinishda bo‘lsa $d(x)=Ax$ chiziqli akslantirish differensiallashdan iboratligini ko‘rsataylik.

$$xy=(x_1e_1+x_2e_2+x_3e_3)(y_1e_1+y_2e_2+y_3e_3)=¿$$

$$¿x_1y_1e_2+x_2y_1e_3$$

tenglikka ko‘ra

$$A(xy)=\begin{pmatrix} a_{1,1} & 0 & 0 \\ a_{2,1} & 2a_{1,1} & 0 \\ a_{3,1} & a_{2,1} & 3a_{1,1} \end{pmatrix}\begin{pmatrix} 0 \\ x_1y_1 \\ x_2y_1 \end{pmatrix}=2a_{1,1}x_1y_1e_2+(a_{2,1}+3a_{1,1})x_2y_1e_3$$

boshqa tamondan

$$Ax=\begin{pmatrix} a_{1,1} & 0 & 0 \\ a_{2,1} & 2a_{1,1} & 0 \\ a_{3,1} & a_{2,1} & 3a_{1,1} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=a_{1,1}x_1e_1+(a_{2,1}x_1+2a_{1,1}x_2)e_2+¿$$

$$(a_{3,1}x_1+a_{2,1}x_2+3a_{1,1}x_3)e_3$$

$$(Ax)y=(a_{1,1}x_1e_1+(a_{2,1}x_1+2a_{1,1}x_2)e_2+(a_{3,1}x_1+a_{2,1}x_2+3a_{1,1}x_3)e_3)$$

$$(y_1e_1+y_2e_2+y_3e_3)=a_{1,1}x_1y_1e_2+(a_{2,1}x_1+2a_{1,1}x_2)y_1e_3$$

$$Ay=\begin{pmatrix} a_{1,1} & 0 & 0 \\ a_{2,1} & 2a_{1,1} & 0 \\ a_{3,1} & a_{2,1} & 3a_{1,1} \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}=a_{1,1}y_1e_1+(a_{2,1}y_1+2a_{1,1}y_2)e_2+¿$$

$$(a_{3,1}y_1+a_{2,1}y_2+3a_{1,1}y_3)e_3$$

$$x(Ay)=(x_1e_1+x_2e_2+x_3e_3)¿$$

$$(a_{3,1}y_1+a_{2,1}y_2+3a_{1,1}y_3)e_3¿=a_{1,1}x_1y_1e_2+a_{1,1}x_2y_1e_3$$

Yuqoridagi xisoblashlar (4) tenglik to‘g‘riligini ko‘rsatadi.

Teorema isbotlandi.

Endi lokal differensiallashni ko‘ramiz

2-Teorema. Agar Δ matritsasi pastki uchburchak shakldа bo‘lsa, N_3 dagi Δ chiziqli akslantirish lokal differensiallash bo`ladi.

Isboti. Δ- N_3bo‘yicha ixtiyoriy lokal differensiallashi bo‘lsin va Δ matritsasi

$$A=\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

bo‘lsin. Barcha $x=x_1e_1+x_2e_2+x_3e_3$ uchun ta’rifga ko‘ra N_3 ustida Adifferensiallash mavjudki

$$\Delta(x)=A_x(x)$$

tenglik o`rinli.

2-teoremaga ko`ra $A_x$ differensiallash quyidagi ko`rinishga ega bo`lsin

$$A_x=\begin{pmatrix} \alpha_x & 0 & 0 \\ \beta_x & 2\alpha_x & 0 \\ \gamma_x & \beta_x & 3\alpha_x \end{pmatrix}.$$

$x=e_1, x=e_2, x=e_3$larni tanlab va $\Delta(x)=A_x(x)$ tenglikdan foydalanib, ya`ni $A\bar{x}=A_x\bar{x}$ ga ko`ra $b_{12}=b_{13}=b_{23}=0$ natijaga ega bo`lamiz. Bu yerda $\bar{x}-x$ ga mos vektor. Bundan A matritsa

$$A=\begin{pmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

ko`rinishga keladi.

Yana $\nabla\ (x)=A_x(x)$ dan foydalanib, ya’ni $A\bar{x}=A_x\bar{x}$ ga ko`ra

$$\begin{pmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} \alpha_x & 0 & 0 \\ \beta_x & 2\alpha_x & 0 \\ \gamma_x & \beta_x & 3\alpha_x \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Tenglamalar sistemasiga ega bo`lamiz.

$$\begin{cases} \alpha_x x_1=b_{1,1}x_1 \\ \beta_x x_1+2\alpha_x x_2=b_{2,1}\cdot x_1+b_{2,2}\cdot x_2 \\ \gamma_x x_1+\beta_x x_2+3\alpha_x x_3=b_{3,1}x_1+b_{3,2}x_2+b_{3,3}x_3 \end{cases}$$

Agar $x_1\neq 0$bo‘lsa

$$\alpha_x=b_{1,1}$$

$$\beta_x=b_{2,1}+(b_{2,2}-2\alpha_x)\frac{x_2}{x_1}$$

$$\gamma_x=b_{3,1}+(b_{3,2}-\beta_x)\frac{x_2}{x_1}+(b¿¿3,3-3\alpha_x)\frac{x_3}{x_1}$$
¿

Agar $x_1=0, x_2\neq 0$ bo‘lsa,

$$2\alpha_x=b_{2,2},$$

$$\beta_x=b_{3,2}+(b¿¿3,3-3\alpha_x)\frac{x_3}{x_1}¿$$

Agar $x_1=x_2=0, x_3\neq 0$ bo‘lsa,

$$3\alpha_x=b_{3,3}.$$

Teorema isbotlandi.

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